CS540 Introduction to Artificial Intelligence

Lecture 22

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Traveler's Dilemma

Quiz

Traveler's Dilemma, Rationalizability

Quiz

Summary

Discussion

Guess Average Game

Motivation

Guess Average Game Derivation

Motivation

Rationalizability

Motivation

- An action is 1-rationalizable if it is the best response to some action.
- An action is 2-rationalizable if it is the best response to some 1-rationalizable action.
- An action is 3-rationalizable if it is the best response to some 2-rationalizable action.
- An action is rationalizable if it is ∞-rationalizable.

Rationalizability Example

Quiz

Best Response

Definition

- An action is a best response if it is optimal for the player given the opponents' actions.

$$br_{MAX}\left(s_{MIN}\right) = \underset{s \in S_{MAX}}{\operatorname{argmax}}\, c\left(s, s_{MIN}\right)$$

$$br_{MIN}\left(s_{MAX}\right) = \underset{s \in S_{MIN}}{\operatorname{argmin}}\, c\left(s_{MAX}, s\right)$$

Nash Equilibrium

Definition

- A Nash equilibrium is a state in which all actions are best responses.

Nash Equilibrium Example 1

Quiz

Nash Equilibrium Example 1

Quiz

Nash Equilibrium Example 2

Quiz

Prisoner's Dilemma

Discussion

- A simultaneous move, non-zero-sum, and symmetric game is a prisoner's dilemma game if the Nash equilibrium state is strictly worse for both players than another state.

–	C	D
C	(x, x)	$(0, y)$
D	$(y, 0)$	$(1, 1)$

- C stands for Cooperate and D stands for Defect (not Confess and Deny). Both players are MAX players. The game is PD if $y > x > 1$. Here, (D, D) is the only Nash equilibrium and (C, C) is strictly better than (D, D) for both players.

Prisoner's Dilemma Derivation

Discussion

Properties of Nash Equilibrium

Discussion

- All Nash equilibria are rationalizable.
- No Nash equilibrium contains a strictly dominated action.
- Rationalizable actions (the set of Nash equilibria is a subset of this) can be found be iterated elimination of strictly dominated actions.
- The above statements are not true for weakly dominated actions.

Mixed Strategy Nash Equilibrium

Definition

- A mixed strategy is a strategy in which a player randomizes between multiple actions.
- A pure strategy is a strategy in which all actions are played with probabilities either 0 or 1.
- A mixed strategy Nash equilibrium is a Nash equilibrium for the game in which mixed strategies are allowed.

Rock Paper Scissors Example

Discussion

Rock Paper Scissors Example Derivation

Discussion

Battle of the Sexes Example

Quiz

Battle of the Sexes Example 1

Quiz

Battle of the Sexes Example 1 Derivation 1

Quiz

Nash Theorem

Definition

- Every finite game has a Nash equilibrium.
- The Nash equilibria are fixed points of the best response functions.

Summary

Discussion