CS540 Introduction to Artificial Intelligence

Lecture 22

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Based on lecture slides by Jerry Zhu, Yingyu Liang, and Charles Dyer

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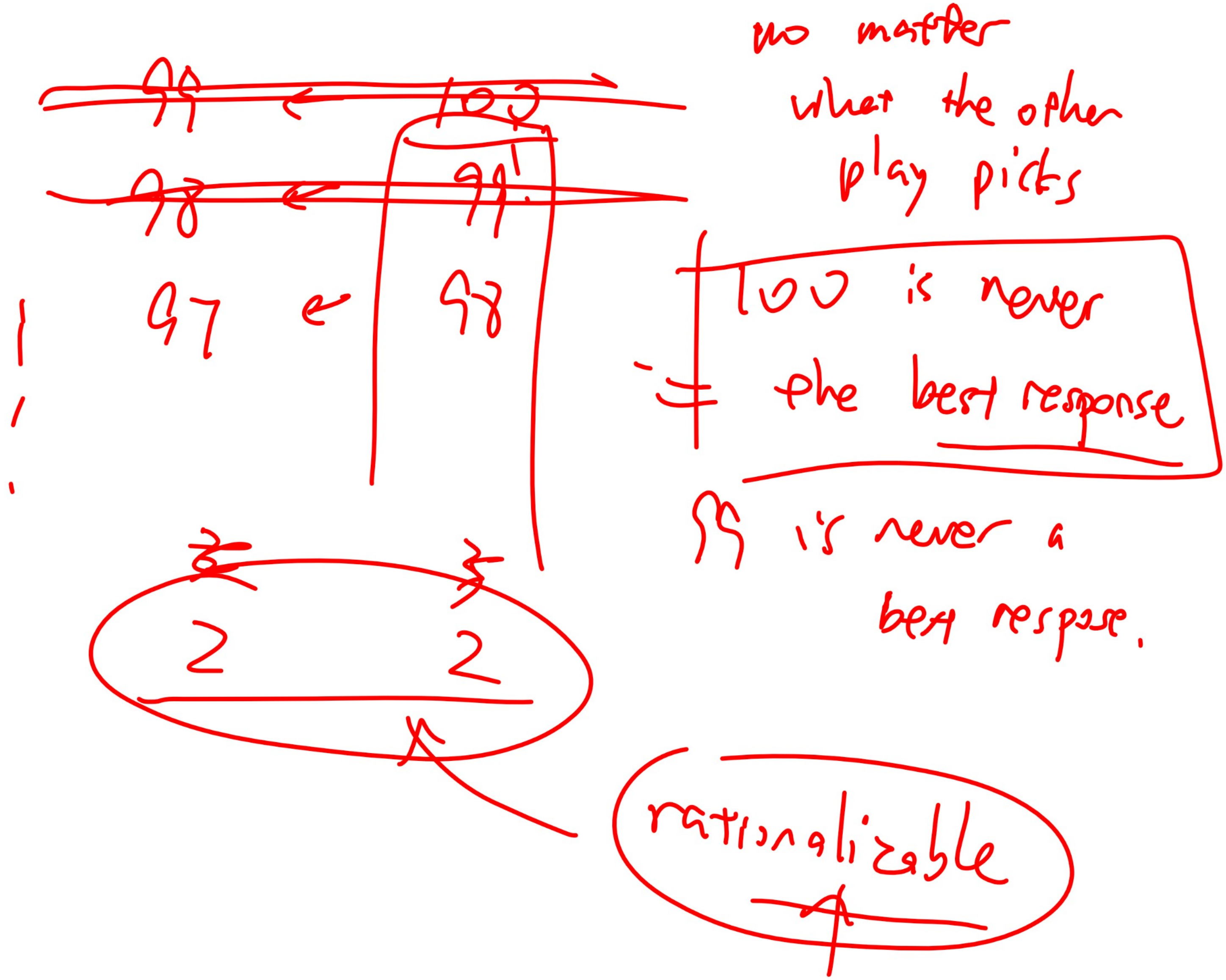
Traveler's Dilemma

Quiz

- Two identical antiques are lost. The airline only knows that its value is at most 100 dollars, so the airline asks their owners (travelers) to report its value (non-negative integers, $\geqslant 2$). The airline tells the travelers that they will be paid the minimum of the two reported values, and the traveler who reported a strictly lower value will receive 2 dollars in reward. If you are one of the travelers, what will you report?

Traveler's Dilemma, Rationalizability

Quiz



Summary

Discussion

- Adversarial Search:

1. Sequential Move Games: Minimax $\rightarrow$ DFS on the game tree.
2. Sequential Move Games: Alpha-Beta Pruning $\rightarrow$ DFS to keep track α and β $\rightarrow$ prune the subtree with $\alpha \Rightarrow \beta$.
3. Simultaneous Move Games: Iterated Elimination of Strictly Dominated Strategies (Rationalizability).
4. Simultaneous Move Games: Nash Equilibrium.

Guess Average Game

Motivation

- Write down an integer between 0 and 100 that is the closest to two thirds (2/3) of the average of everyone's (including yours) integers.

$\frac{2}{3} \cdot 100 \approx 66$

any ≥ 66 → never best action

all players will pick 0 → 66

any ≥ 44 → never best

Guess Average Game Derivation

Motivation

[0, 1] → rationalizable. ←

Rationalizability

Motivation

- An action is 1-rationalizable if it is the best response to some action.
- An action is 2-rationalizable if it is the best response to some 1-rationalizable action.
- An action is 3-rationalizable if it is the best response to some 2-rationalizable action.
- An action is rationalizable if it is ∞-rationalizable.

Rationalizability Example

Quiz

- Both players are MAX players. Which actions are rationalizable for the ROW player?

–	A	B	C
A	(2,4)	(3,7)	(4,5)
B	(1,2)	(5,4)	(2,3)
C	(4,1)	(2,8)	(5,3)
D	(3,6)	(4,0)	(1,9)

Q7

COL

ROW

A, B, C,

B, C, C.

Solution of game

Best Response

Definition

- An action is a best response if it is optimal for the player given the opponents' actions.

$$br_{MAX}\left(s_{MIN}\right) = \underset{s \in S_{MAX}}{\operatorname{argmax}}\, c\left(s, s_{MIN}\right)$$

$$br_{MIN}\left(s_{MAX}\right) = \underset{s \in S_{MIN}}{\operatorname{argmin}}\, c\left(s_{MAX}, s\right)$$

Nash Equilibrium

Definition

- A Nash equilibrium is a state in which all actions are best responses.

Nash Equilibrium Example 1

Quiz

- Find the value of the Nash equilibrium of the following zero-sum game.

MIN

MAX

—	I	II	III
I	-4	-7	-3
II	9	1	7
III	-6	-1	5

Nash equilibrium

Nash Equilibrium Example 1

Quiz

- Find the value (of MAX player) of the Nash equilibrium of the following zero-sum game.

—	I	II	III
I	(-4, 4)	(-7, 7)	(-3, 3)
II	(9, −9)	(1, −1)	(7, −7)
III	(-6, 6)	(-1, 1)	(5, −5)

Nash Equilibrium Example 2

Quiz

- Find the value of the Nash equilibrium of the following zero-sum game.

—	I	II	III
I	1	2	3
II	4	5	6
III	7	8	9

- A: 1 , B: 3 , C: 5 , D: 7, E: I don't understand

Prisoner's Dilemma

Discussion

- A simultaneous move, non-zero-sum, and symmetric game is a prisoner's dilemma game if the Nash equilibrium state is strictly worse for both players than another state.

—	C	D
C	(x, x)	$(0, y)$
D	$(y, 0)$	$(1, 1)$

- C stands for Cooperate and D stands for Defect (not Confess and Deny). Both players are MAX players. The game is PD if $y > x > 1$. Here, (D, D) is the only Nash equilibrium and (C, C) is strictly better than (D, D) for both players.

Prisoner's Dilemma Derivation

Discussion

Properties of Nash Equilibrium

Discussion

- All Nash equilibria are rationalizable.
- No Nash equilibrium contains a strictly dominated action.
- Rationalizable actions (the set of Nash equilibria is a subset of this) can be found be iterated elimination of strictly dominated actions.
- The above statements are not true for weakly dominated actions.

Mixed Strategy Nash Equilibrium

Definition

- A mixed strategy is a strategy in which a player randomizes between multiple actions.
- A pure strategy is a strategy in which all actions are played with probabilities either 0 or 1.
- A mixed strategy Nash equilibrium is a Nash equilibrium for the game in which mixed strategies are allowed.

Rock Paper Scissors Example

Discussion

- There are no pure strategy Nash equilibria.
- Playing each action (rock, paper, scissors) with equal probability is a mixed strategy Nash.

Rock Paper Scissors Example Derivation

Discussion

Battle of the Sexes Example

Quiz

- Battle of the Sexes (BoS, also called Bach or Stravinsky) is a game that models coordination in which two players have different preferences in which alternative to coordinate on.

–	Bach	Stravinsky
Bach	(x, y)	$(0, 0)$
Stravinsky	$(0, 0)$	(y, x)

Battle of the Sexes Example 1

Quiz

- Find all Nash equilibria of the following game.

–	I	II
I	(3, 5)	(0, 0)
II	(0, 0)	(5, 3)

5/8 3/8

p

1−p

q 1−q

$q = \frac{5}{8}$

Row:

I : $3 \cdot q$

II : $5(1-q)$

BR

I $3q > 5(1-q)$

any mix $3q = 5(1-q)$

II $5(1-q) > 3q$

Battle of the Sexes Example 1 Derivation 1

Quiz

COL: I: $5p$

II: $3(1-p)$

BR: I if $5p > 3(1-p)$; mix if $5p = 3(1-p)$; II if $5p < 3(1-p)$

—	I	II
I	(3, 5)	(0, 0)
II	(0, 0)	(5, 3)

Rows: I: $p = \frac{3}{8}$; II: $1-p = \frac{5}{8}$

Columns: I: $\frac{5}{8}$; II: $\frac{3}{8}$

$p = \frac{3}{8}$

Nash Theorem

Definition

- Every finite game has a Nash equilibrium.
- The Nash equilibria are fixed points of the best response functions.

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Discussion

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1. Sequential Move Games: Minimax $\rightarrow$ DFS on the game tree.
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3. Simultaneous Move Games: Iterated Elimination of Strictly Dominated Strategies (Rationalizability) $\rightarrow$ Remove dominated actions for each player $\rightarrow$ Repeat.
4. Simultaneous Move Games: Nash Equilibrium $\rightarrow$ Compute the best response $\rightarrow$ Find strategies (pure or mixed) that are mutual best responses.