

Bayes Net Training Example, Training Quiz

- Given a network and the training data.
 $H \rightarrow B, D \rightarrow B, B \rightarrow J, B \rightarrow A.$

H	D	B	J	A
0	0	0	1	0
0	0	0	0	1
1	0	0	1	1
0	1	0	1	1
0	0	1	1	0
0	0	1	0	1
1	0	1	1	0
0	1	1	1	0

Bayes Net Training Example, Training 1

Quiz

- Compute $\hat{\mathbb{P}}\{D = 1\}$.

<i>H</i>	<i>D</i>	<i>B</i>	<i>J</i>	<i>A</i>
0	0	0	1	0
0	0	0	0	1
1	0	0	1	1
0	1	0	1	1
0	0	1	1	0
0	0	1	0	1
1	0	1	1	0
0	1	1	1	0

Bayes Net Training Example, Training 2

Quiz

- Compute $\hat{\mathbb{P}}\{J = 1|B = 1\}$.

H	D	B	J	A
0	0	0	1	0
0	0	0	0	1
1	0	0	1	1
0	1	0	1	1
0	0	1	1	0
0	0	1	0	1
1	0	1	1	0
0	1	1	1	0

Bayes Net Training Example, Training 3

Quiz

- What is the conditional probability $\hat{\mathbb{P}}\{J = 1|B = 0\}$?
- A: I don't understand, B: $\frac{1}{4}$, C: $\frac{1}{2}$, D: $\frac{3}{4}$, E: 1

<i>H</i>	<i>D</i>	<i>B</i>	<i>J</i>	<i>A</i>
0	0	0	1	0
0	0	0	0	1
1	0	0	1	1
0	1	0	1	1
0	0	1	1	0
0	0	1	0	1
1	0	1	1	0
0	1	1	1	0

Bayes Net Training Example, Training 4

Quiz

- Compute $\hat{\mathbb{P}}\{B = 1 | H = 0, D = 1\}$.

H	D	B	J	A
0	0	0	1	0
0	0	0	0	1
1	0	0	1	1
0	1	0	1	1
0	0	1	1	0
0	0	1	0	1
1	0	1	1	0
0	1	1	1	0

Bayes Net Training Example, Training 5

Quiz

- What is the conditional probability $\hat{\mathbb{P}}\{B = 1 | H = 0, D = 0\}$?
- A: I don't understand, B: $\frac{1}{4}$, C: $\frac{1}{2}$, D: $\frac{3}{4}$, E: 1

H	D	B	J	A
0	0	0	1	0
0	0	0	0	1
1	0	0	1	1
0	1	0	1	1
0	0	1	1	0
0	0	1	0	1
1	0	1	1	0
0	1	1	1	0

Bayes Net Training Example, Training 5

Quiz

- What is the conditional probability $\hat{\mathbb{P}}\{A = 0 | H = 1, D = 1\}$?
- A : I don't understand, B: 0 , C: $\frac{1}{2}$, D: 1 , E: NA

<i>H</i>	<i>D</i>	<i>B</i>	<i>J</i>	<i>A</i>
0	0	0	1	0
0	0	0	0	1
1	0	0	1	1
0	1	0	1	1
0	0	1	1	0
0	0	1	0	1
1	0	1	1	0
0	1	1	1	0

Laplace Smoothing

Definition

- Recall that the MLE estimation can incorporate Laplace smoothing.

$$\hat{\mathbb{P}}\{x_j | p(X_j)\} = \frac{c_{x_j, p(X_j)} + 1}{c_p(X_j) + |X_j|}$$

- Here, $|X_j|$ is the number of possible values (number of categories) of X_j .
- Laplace smoothing is considered regularization for Bayesian networks because it avoids overfitting the training data.

Bayes Net Inference 1

Definition

- Given the conditional probability table, the joint probabilities can be calculated using conditional independence.

$$\begin{aligned}\mathbb{P}\{x_1, x_2, \dots, x_m\} &= \prod_{j=1}^m \mathbb{P}\{x_j | x_1, x_2, \dots, x_{j-1}, x_{j+1}, \dots, x_m\} \\ &= \prod_{j=1}^m \mathbb{P}\{x_j | p(x_j)\}\end{aligned}$$

Bayes Net Inference 2

Definition

- Given the joint probabilities, all other marginal and conditional probabilities can be calculated using their definitions.

$$\mathbb{P}\{x_j | x_{j'}, x_{j''}, \dots\} = \frac{\mathbb{P}\{x_j, x_{j'}, x_{j''}, \dots\}}{\mathbb{P}\{x_{j'}, x_{j''}, \dots\}}$$

$$\mathbb{P}\{x_j, x_{j'}, x_{j''}, \dots\} = \sum_{x_k: k \neq j, j', j'', \dots} \mathbb{P}\{x_1, x_2, \dots, x_m\}$$

$$\mathbb{P}\{x_{j'}, x_{j''}, \dots\} = \sum_{x_k: k \neq j', j'', \dots} \mathbb{P}\{x_1, x_2, \dots, x_m\}$$

Bayes Net Inference Example 1

Quiz

- Assume the network is trained on a larger set with the following CPT. Compute $\hat{\mathbb{P}}\{H = 0, D = 1 | J = 1, A = 0\}$?

$$\hat{\mathbb{P}}\{H = 1\} = 0.001, \hat{\mathbb{P}}\{D = 1\} = 0.001$$

$$\hat{\mathbb{P}}\{B = 1 | H = 1, D = 1\} = 0.95, \hat{\mathbb{P}}\{B = 1 | H = 1, D = 0\} = 0.94$$

$$\hat{\mathbb{P}}\{B = 1 | H = 0, D = 1\} = 0.29, \hat{\mathbb{P}}\{B = 1 | H = 0, D = 0\} = 0.00$$

$$\hat{\mathbb{P}}\{J = 1 | B = 1\} = 0.9, \hat{\mathbb{P}}\{J = 1 | B = 0\} = 0.05$$

$$\hat{\mathbb{P}}\{A = 1 | B = 1\} = 0.7, \hat{\mathbb{P}}\{A = 1 | B = 0\} = 0.01$$

Bayes Net Inference Example 1 Computation 1

Quiz

Bayes Net Inference Example 1 Computation 2

Quiz

Bayes Net Inference Example 2

Quiz

- Compute $\hat{\mathbb{P}}\{D = 1|H = 0\}$?

$$\hat{\mathbb{P}}\{H = 1\} = 0.001, \hat{\mathbb{P}}\{D = 1\} = 0.001$$

$$\hat{\mathbb{P}}\{B = 1|H = 1, D = 1\} = 0.95, \hat{\mathbb{P}}\{B = 1|H = 1, D = 0\} = 0.94$$

$$\hat{\mathbb{P}}\{B = 1|H = 0, D = 1\} = 0.29, \hat{\mathbb{P}}\{B = 1|H = 0, D = 0\} = 0.00$$

- A : 0, B: 0.001, C: 0.0094, D: 0.0095, E: 1

Bayes Net Inference Example 2 Derivation

Quiz

Bayes Net Inference Example 3

Quiz

- Compute $\hat{\mathbb{P}}\{H = 0, D = 1 | B = 1\}$?

$$\hat{\mathbb{P}}\{H = 1\} = 0.001, \hat{\mathbb{P}}\{D = 1\} = 0.001$$

$$\hat{\mathbb{P}}\{B = 1 | H = 1, D = 1\} = 0.95, \hat{\mathbb{P}}\{B = 1 | H = 1, D = 0\} = 0.94$$

$$\hat{\mathbb{P}}\{B = 1 | H = 0, D = 1\} = 0.29, \hat{\mathbb{P}}\{B = 1 | H = 0, D = 0\} = 0.00$$

- A : 0, B: 0.001, C: 0.0094, D: 0.0095, E: 1

Bayes Net Inference Example 3 Derivation

Quiz

Bayes Net Inference Example 4

Quiz

- Compute $\hat{\mathbb{P}}\{B = 1|J = 1, A = 0\}$?

$$\hat{\mathbb{P}}\{J = 1|B = 1\} = 0.9, \hat{\mathbb{P}}\{J = 1|B = 0\} = 0.05$$

$$\hat{\mathbb{P}}\{A = 1|B = 1\} = 0.7, \hat{\mathbb{P}}\{A = 1|B = 0\} = 0.01$$

Given

$$\mathbb{P}\{B = 1\} = 0.001 \cdot 0.001 \cdot 0.95 + 0.001 \cdot 0.999 \cdot (0.94 + 0.29).$$

- A : 0, B: 0.001, C: 0.0094, D: 0.0095, E: 1

Bayes Net Inference Example 4 Derivation

Quiz

