X7 Q1

Question 1

- [3 points] Given the variance matrix $\hat{\Sigma}$ is a diagonal matrix, what is the smallest value of K so that the Manhattan distance between the vector $\begin{bmatrix} 1 \\ 1 \\ \cdots \\ 1 \end{bmatrix}$ with 38 1's and its reconstruction using the first K principal components is less than or equal to 8?

$30 = \boxed{\text{smallest value of } K}$

38×38

$PC_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$, $PC_i = \begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix}$

all PCs look like this.

$\hat{\Sigma} = \begin{bmatrix} 1 & & & & 0 \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ 0 & & & & 1 \end{bmatrix} \begin{bmatrix} 1 & & & & 0 \\ & 2 & & & \\ & & 3 & & \\ & & & 4 & \\ 0 & & & & 5 \end{bmatrix} \begin{bmatrix} 1 & & & & 0 \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ 0 & & & & 1 \end{bmatrix}$

original features ; new feature

$38 \left\{ \begin{pmatrix} | \\ | \\ | \\ | \end{pmatrix} \right. = X$

$x' = \begin{pmatrix} PC_1 \cdot X \\ PC_2 \cdot X \\ \vdots \\ PC_K \cdot x \end{pmatrix} = \left. \begin{pmatrix} | \\ | \\ | \end{pmatrix} \right\} K$

reconstruction

q

$$\underbrace{38}_{d} \left\{ \begin{pmatrix}0\\1\\0\\0\\1\\0\\0\end{pmatrix} \right. \approx \hat{X} = X'_1 \cdot \underline{PC_1} + X'_2 \cdot \underline{PC_2} + \dots \; X'_k \, \underline{PC_k}$$

$$= \begin{pmatrix}0\\1\\0\\0\\1\\0\\1\end{pmatrix} \begin{array}{l} \text{K of them are 1} \\ \text{38-K of them are 0} \end{array}$$

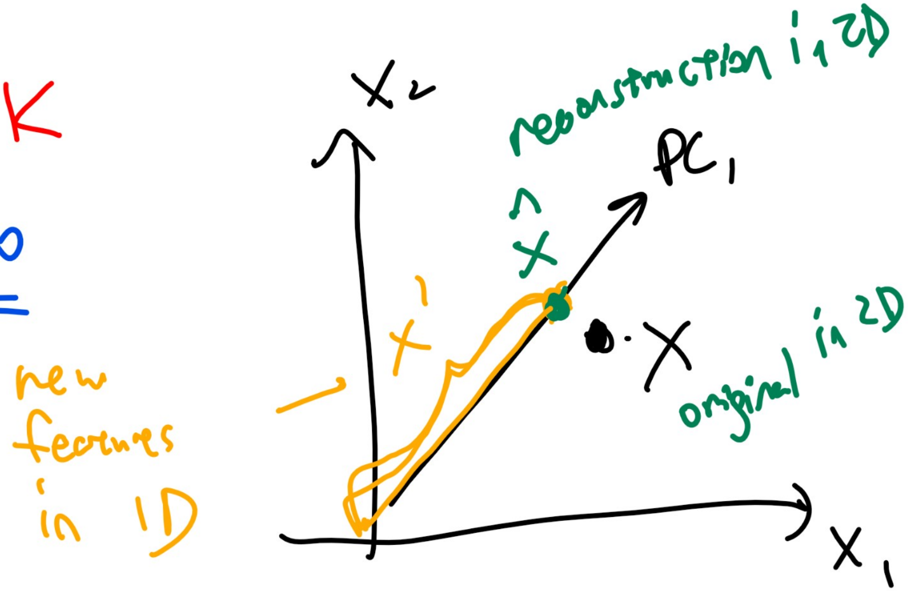
K 1's

38 - K 0's

$$d\left(\begin{pmatrix}1\\1\\1\\1\end{pmatrix}, \begin{pmatrix}0\\0\\0\\0\\1\\0\\0\end{pmatrix} \right) = (38-K)\cdot(1-0) + K\cdot(1-1)$$

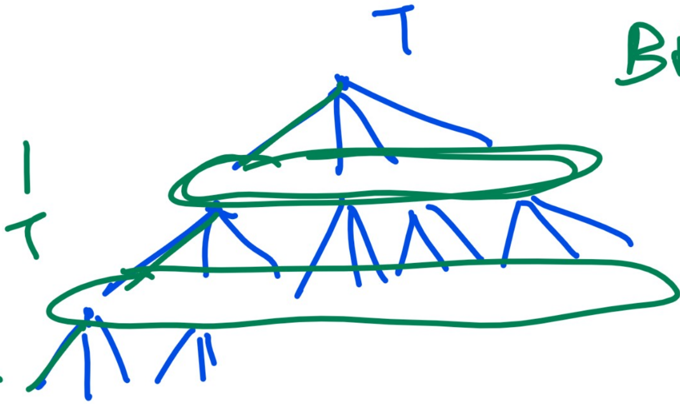
$$= 38 - K$$

want $38 - K \le 8$ $\quad \underline{\underline{K \geq 30}}$



Question 2

- [3 points] In a 200 by 140 grid, Tom is located at $(98, 54)$ and Jerry is located at $(112, 95)$. Tom uses BFS (Breadth First Search) to find Jerry and the successors of a state (one cell in the grid) are the four neighboring states on the grid (the cells above, below, to the left and to the right). What is the minimum number of states that need to be expanded to find (and expand) the goal state? The order in which the successors are added can be arbitrary. Include both the initial and the goal states.
- Answer: ______ . Calculate

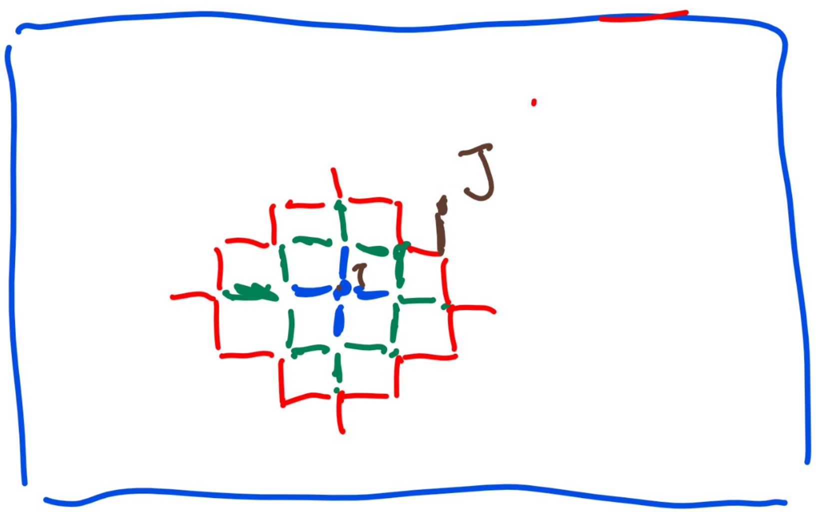


BFS

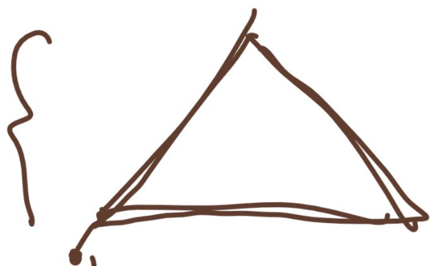
dist 1 from T

dist 2 from T

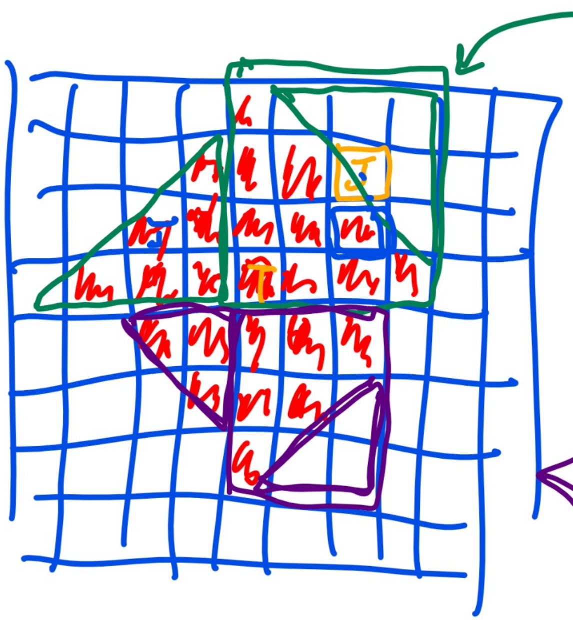
Manhattan distance



dist ≤ 3



dist = 4 goal



$d = 4$

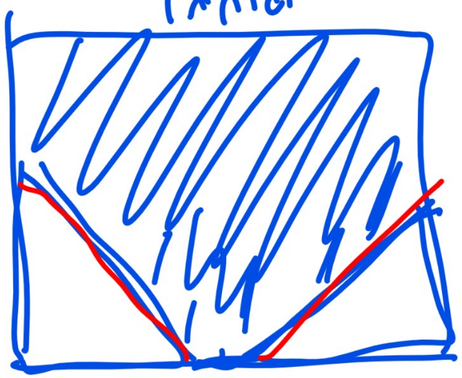
answer =

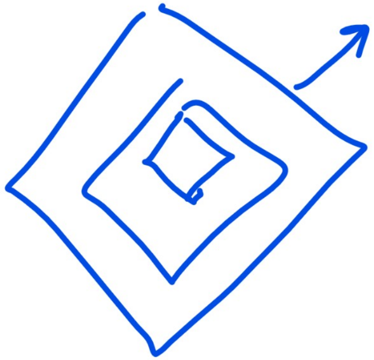
$$d^2 + (d-1)^2 + 1$$

J goal state

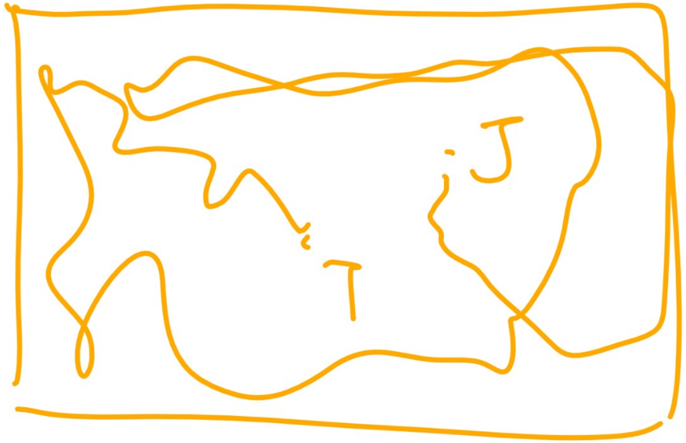
$$d = \underbrace{|112 - 98| + |95 - 54|}_{\text{Manhattan dist between T and J.}}$$

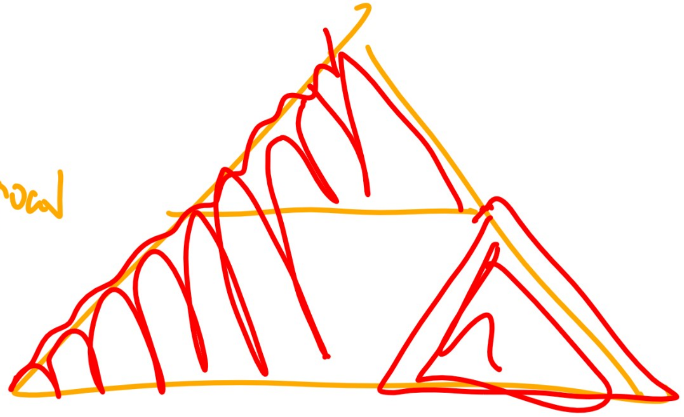
P5



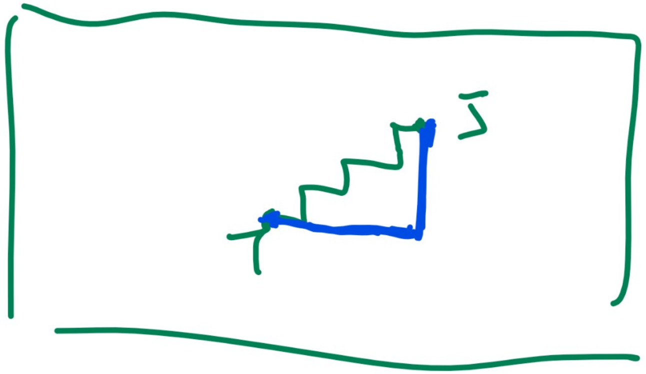


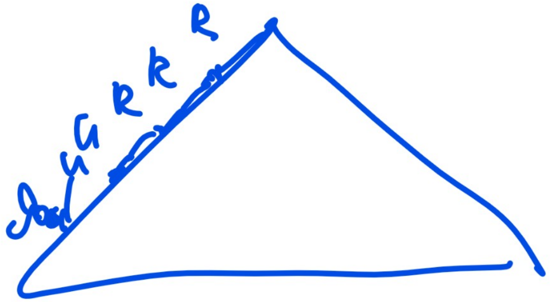
worst





best case





expand goal state
to stop

DFS = $\underline{d}$

Manhattan dist.

$\underline{\text{BFS}}$, A^* → stop after goal is expanded.

Question 3

- [3 points] There are $n = 101$ students in CS540, for simplicity, assume student 0 gets grade $g = 0$, student 1 gets grade $g = 1$, ..., student $n - 1$ gets grade $g = n - 1$. The payoff (value) for each student who drop the course is 0, the payoff for the students who stay is $0.02\ \mathrm{g} - 1.5$ if the student has the lowest grade among all students who decide to stay in the class, and the $0.02\ \mathrm{g} - 1$ otherwise. If each student only uses actions that are rationalizable (i.e. survive the iterated elimination of strictly dominated actions), how many students will stay in the course? If there are multiple correct answers, enter one of them.
- Answer: [ ] . Calculate

$g \leq 75$ D

$g \geq 75$ S

rational → do not play action never best response.
do not play strictly dominated action.
no matter what other players are doing, this action is worse.

① even if I have lowest grade I would not drop
② even if I do not have lowest grade, I would drop.

① $0.02g - 1.5 > 0 \Rightarrow g > 75 \Rightarrow$ rational action is Stay.

② $0.02g - 1 < 0 \Rightarrow g < 50 \Rightarrow$ rational action is Drop.

$\underbrace{50 \leq g \leq 75}$ $\Rightarrow$ rational action : Stay, Drop.

rationalizable : Drop.

$g = 50$, assume other students are rational

$g < 50$ drop $\Rightarrow$ $g=50$ is definitely lowest

$0.02 \cdot 50 \boxed{-1.5} < 0 \Rightarrow$ Drop.

$g = 51$, assume other students are rational

AND they assume other students are rational

$g < 50$, $\underline{g = 50}$ drop $\Rightarrow$ $g = M$ lowest

$0.02 \cdot 51 \boxed{-1.5} < 0 \Longrightarrow$ Drop.

$\vdots$

$g = 74 \qquad 0.02 \cdot 74 - 1.5 < 0 \Longrightarrow$ Drop.

$g = 75 \qquad \boxed{0.02 \cdot 75 - 1.5 = 0} \Longrightarrow$ Drop / Stay

x

$\underbrace{0, 1, \ldots 49}_{\text{rational D}}, \underbrace{50, 51 \ldots 74}_{\text{rationalizable D}}, \underbrace{75}_{\text{D or S}}, \underbrace{76 \ldots 100}_{\text{rational S}}$

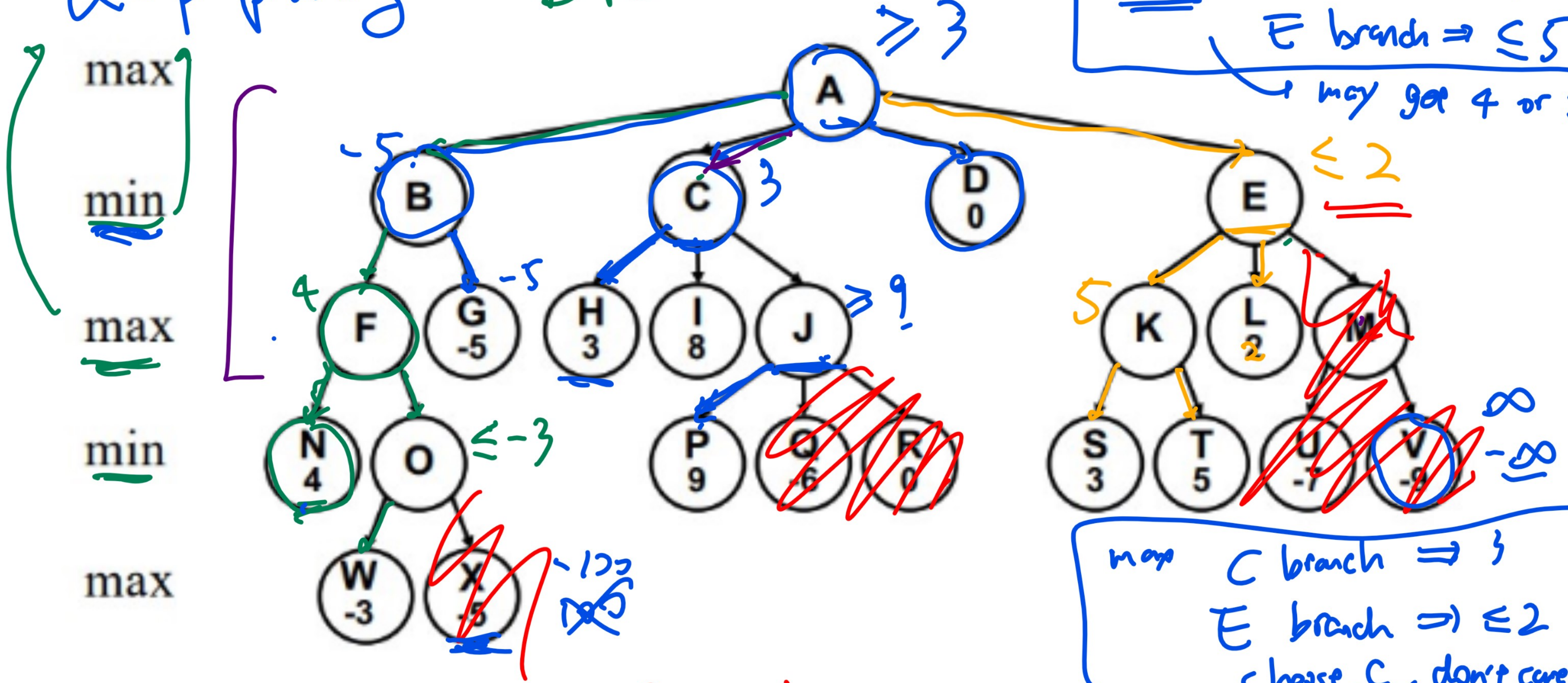
2 rationalizable actions

ans = $\underline{\underline{25}}$ OR $\underline{\underline{26}}$.

2 NE.

α-β pruning DFS

max C branch ⇒ 3
E branch ⇒ ≤ 5
may get 4 or 5



max C branch ⇒ 3
E branch ⇒ ≤ 2
choose C, don't care E.

max can get ≥ 4 from left

max can get ≤ -3 from right

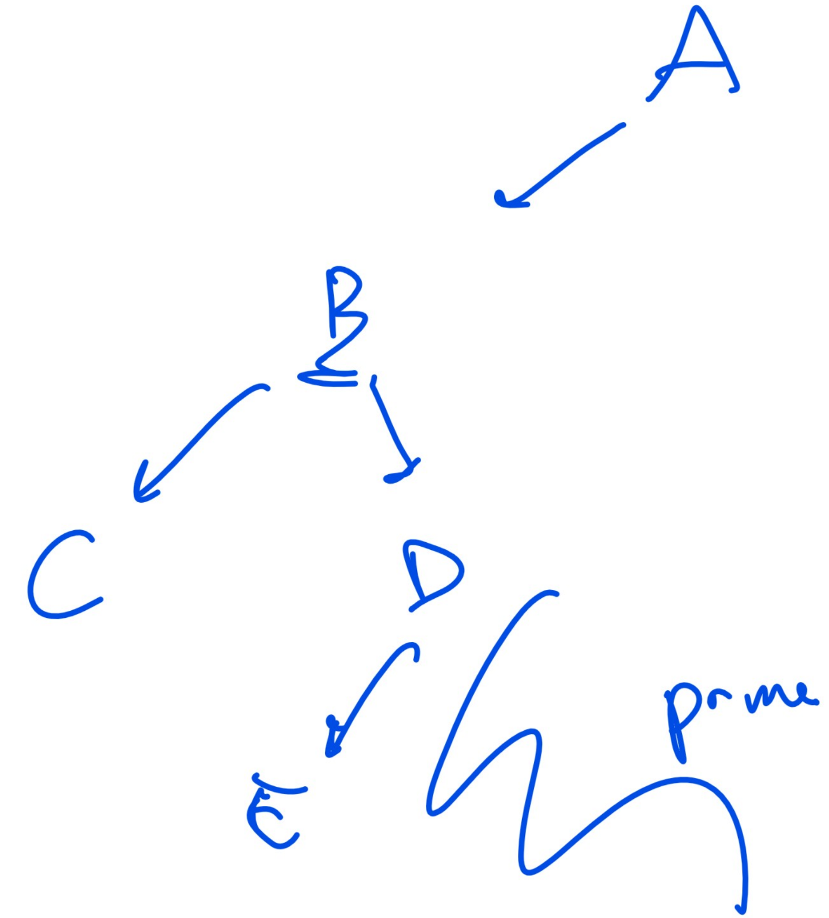
max doesnot care exact value ≤ -3 ⇒ prune.

α-β pruning

pruned total of 6

solution CS ⇒ sequence of move Game ⇒ strategy in all situations

α-β prune $\Longrightarrow$ avoid continue search.



back at 7:10.

beam search not on exam.

Question 10

M11

• [4 points] Given the following game payoff table, suppose the the row player uses a mixed strategy playing U with probability p, and column player uses a pure strategy. What is the smallest and largest value of p in a mixed strategy Nash equilibrium?

Row \ Col	L q	R $1-q$
U p	9, 5	9, 0
D $1-p$	9, 0	0, 10

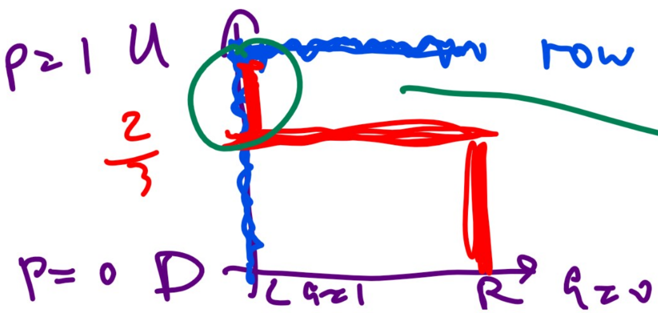
$$BR_{COL} = \begin{cases} L & \text{if } L \geq R \\ \text{mix} & L = R \\ R & L \leq R \end{cases}$$

$5 \cdot p + 0 \cdot (1-p) \geq 0 \cdot p + 10(1-p)$

$p \geq \frac{2}{3}$

$p = \frac{2}{3}$

$p \leq \frac{2}{3}$



mutual best response

$$BR_{row} = \begin{cases} U & \text{if } U \geq D \Rightarrow q \geq q \cdot q + 0(1-q) \\ & \qquad \Rightarrow q \leq 1 \\ \text{mix} & \text{if } U = D \Rightarrow \underbrace{q = 1}_{} \text{ always } L \\ D & \text{if } U \leq D \Rightarrow q = 1 \end{cases}$$

$$\Big(\underbrace{q = 1}_{\text{always } L}, \quad p \in \big[\tfrac{2}{3}, 1 \big] \Big) \Longrightarrow \text{mixed NE}$$

Question 10

• [4 points] Given the following game payoff table, suppose the the row player uses a mixed strategy playing U with probability p, and column player uses a pure strategy. What is the smallest and largest value of p in a mixed strategy Nash equilibrium?

Row \ Col	L	R
U p	9, 5	9, 0
D $1-p$	9, 0	0, 10

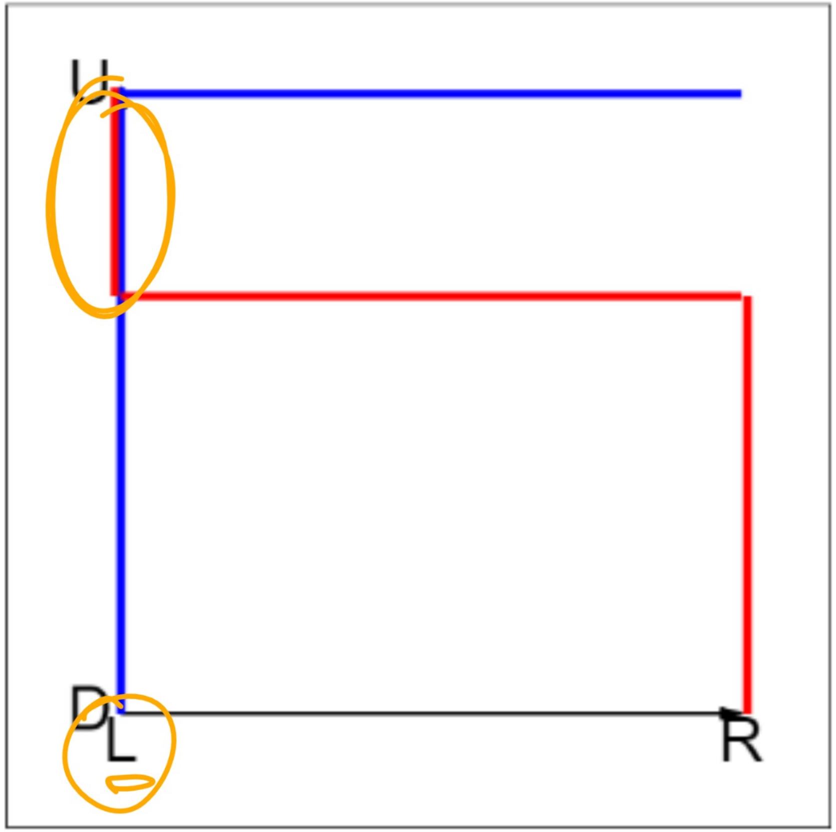
mix

→ if col use R ⇒ row will use U always

⇒ col use L : $L \geqslant R$

$$5p + 0(1-p) \geqslant 0p + 10(1-p)$$

$$p \geqslant \frac{2}{3} \Rightarrow \left[\frac{2}{3}, 1\right]$$



col play L

now play [? , 1]

Question 4

m9

• [2 points] Consider n + 1 = 29 + 1 states. The initial state is 1, the goal state is n. State 0 is a dead-end state with no successors. For each non-0 state i, it has two successors: i + 1 and 0. There is no cycle check nor CLOSED list (this means we may expand (or goal-check) the same nodes many times, because we do not keep track of which nodes are checked previously). How many goal-checks (expanded) will be performed by Breadth First Search? Break ties by expanding the node with the smaller (larger) index first.

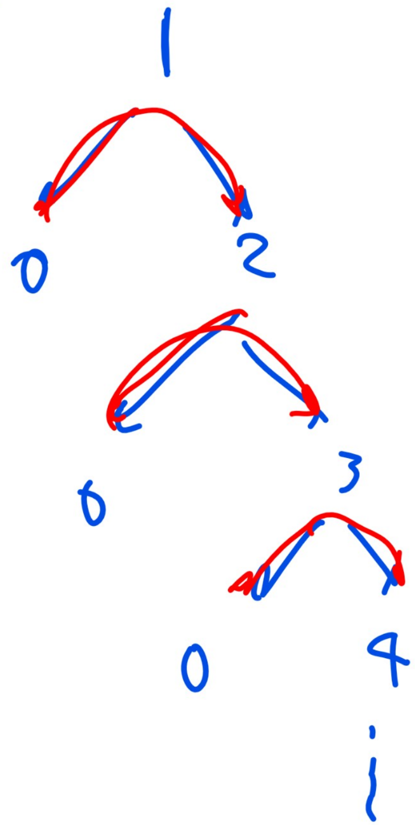
IDS

DFS.

1 0 2 0 3 0 4 ... 0 29

states expanded $= \underbrace{28 \cdot 2 + 1}$ $\Longleftarrow$ BFS

DFS: $\underbrace{1 \quad 0 \quad 2 \quad 0 \quad 3 \;\ldots\; 0 \quad 29}_{28 \cdot 2 + 1}$



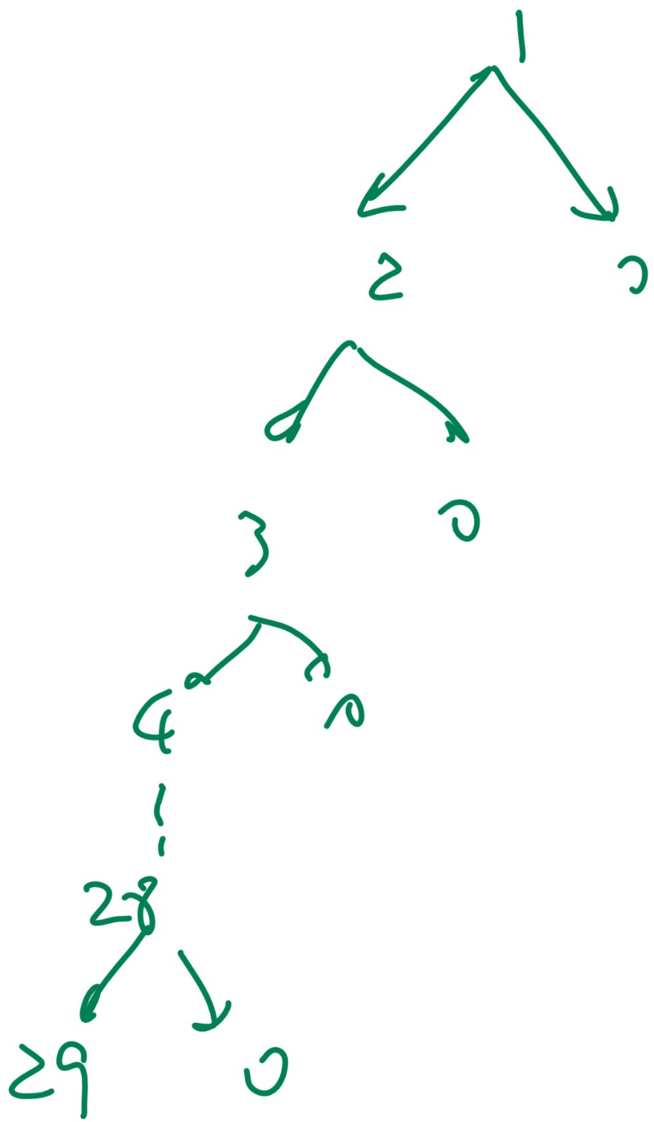
BFS:

Large index first

1 2 0 3 0 4 0

.... 28 0 29

\# states searched = $28 \cdot 2$



DFS: 1, 2, 3 ... 29

\# = 29

IDS $1+3+5+7+\ldots$

Question 15 ×3

- [4 points] Given the dataset $[8 \quad -2 \quad 10 \quad -4 \quad -3]$, the cluster centers are computed by k-means clustering algorithm with $k = 2$. The first cluster center is x and the second cluster center is -7. What is the maximum value of x such that the second cluster is empty (contains 0 instances). In case of a tie in distance, the point belongs to cluster 1.
- Answer: [ ]. Calculate

C_2: -7 $\quad$ $[\,-4 \quad -3 \quad -2 \quad \underset{C_1}{-1} \quad\quad 8 \quad 10\,]$

-4 is "closer" to C_1 compared to C_2,

$\Rightarrow \quad -4 = \frac{1}{2}(C_1 + \cancel{C_2}^{\,-7})$ tie breaking

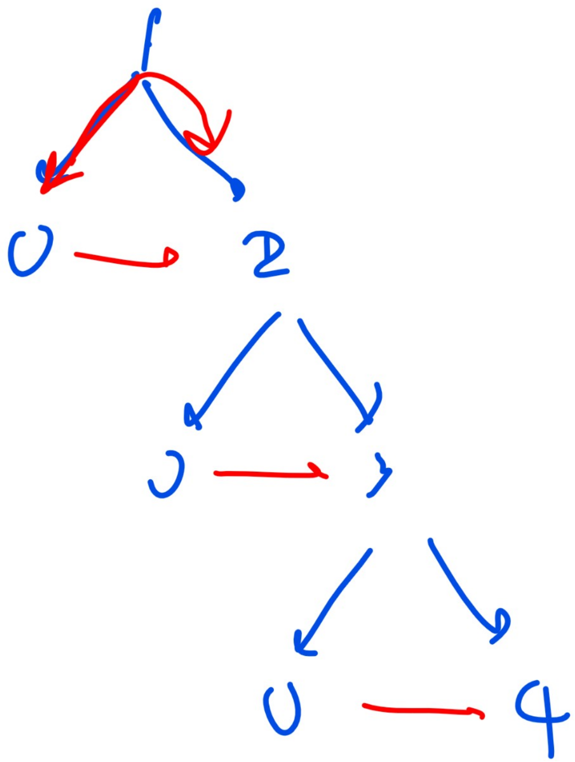
Solve $C_1 = -1$ $\quad\quad$ -4 belongs to C_1

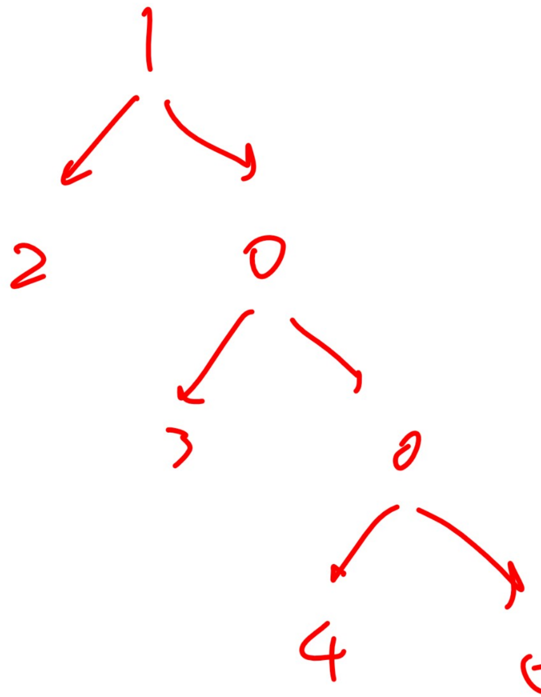
Question 8

• [4 points] When using the Genetic Algorithm, suppose the states are $[1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0]$, $[0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 1]$, $[1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 0]$, $[1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0]$. Let $T = 6$, the fitness function (not the cost) is $\text{argmin}_{t \in \{1,...,T+1\}} x_t = 1$ with $x_{T+1} = 1$ (i.e. the index of the first feature that is 1). What is the reproduction probability of the state with the highest reproduction probability?

• Answer: 1/2 . Calculate

state	fitness	
1	1	1/6
2	3	1/2
3	1	1/6
4	1	1/6

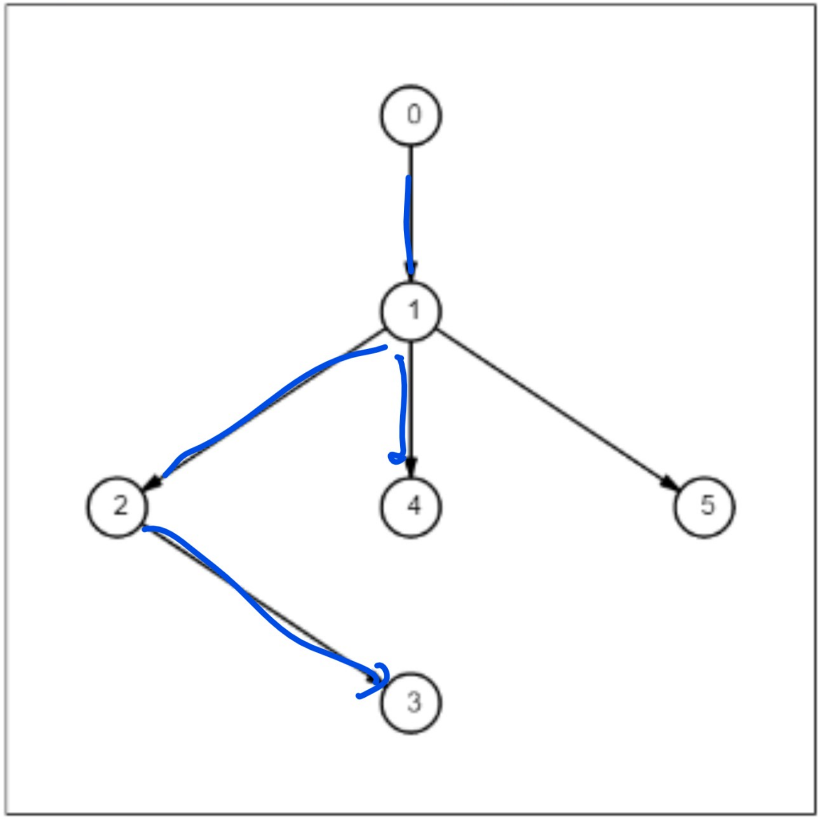




Question 8

• [3 points] Consider Iterative Deepening Search on a tree, where the nodes are denoted by numbers. Write down the sequence IDS visited in the order they are expanded (i.e. expansion path). 0 is the initial state and 5 is the goal state. Start with depth limit 0, include the root, and include repeated nodes.

• Note: use the convention used in the lectures, push the rightmost (in the diagram) successor into the stack first or enqueue the leftmost (in the diagram) successor into the queue first.



expand left ones first

level 0 : 0

level 1 : 0, 1

level 2 : 0, 1, 2, 4, 5

level 3 0, 1, 2, 3, 4, 5