# CS540 Introduction to Artificial Intelligence

## Lecture 11

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# Image Features Diagram

## Motivation

# One Dimensional Convolution

## Definition

- The convolution of a vector $x = (x_1, x_2, ..., x_m)$ with a filter $w = (w_{-k}, w_{-k+1}, ... w_{k-1}, w_k)$ is:

$$a = (a_1, a_2, ..., a_m) = x * w$$

$$a_j = \sum_{t=-k}^{k} w_t x_{j-t}, j = 1, 2, ..., m$$

- $w$ is also called a kernel (different from the kernel for SVMs).
- The elements that do not exist are assumed to be 0.

# Two Dimensional Convolution

## Definition

- The convolution of an $m \times m$ matrix $X$ with a $(2k+1) \times (2k+1)$ filter $W$ is:

$$A = X * W$$

$$A_{j,j'} = \sum_{s=-k}^{k} \sum_{t=-k}^{k} W_{s,t} X_{j-s,j'-t}, j, j' = 1, 2, ..., m$$

- The matrix $W$ is indexed by $(s,t)$ for $s = -k, -k+1, ..., k-1, k$ and $t = -k, -k+1, ..., k-1, k$.
- The elements that do not exist are assumed to be 0.

# Convolution Diagram and Demo

## Definition

# Image Gradient

## Definition

- The gradient of an image is defined as the change in pixel intensity due to the change in the location of the pixel.

$$\frac{\partial I(s,t)}{\partial s} \approx \frac{I\left(s+\frac{\varepsilon}{2},t\right) - I\left(s-\frac{\varepsilon}{2},t\right)}{\varepsilon}, \varepsilon = 1$$

$$\frac{\partial I(s,t)}{\partial t} \approx \frac{I\left(s,t+\frac{\varepsilon}{2}\right) - I\left(s,t-\frac{\varepsilon}{2}\right)}{\varepsilon}, \varepsilon = 1$$

# Image Derivative Filters

## Definition

- The gradient can be computed using convolution with the following filters.

$$w_x = \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}, w_y = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

# Sobel Filter

## Definition

- The Sobel filters also are used to approximate the gradient of an image.

$$W_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}, W_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

# Gradient of Images

## Definition

- The gradient of an image $I$ is $(\nabla_x I, \nabla_y I)$.

$$\nabla_x I = W_x * I, \nabla_y I = W_y * I$$

- The gradient magnitude is $G$ and gradient direction $\Theta$ are the following.

$$G = \sqrt{\nabla_x^2 + \nabla_y^2}$$

$$\Theta = \arctan\left(\frac{\nabla_y}{\nabla_x}\right)$$

# Gradient of Images Demo

## Definition

# Convolution Example

## Quiz

- Find the gradient magnitude and direction for the center cell of the following image. Use the derivative filters $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$
- $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

# Gradient Example

## Quiz

# Convolution Example 1

## Quiz

$$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} * \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

- $A : \begin{bmatrix} -1 & -3 & -3 \\ 0 & 0 & 0 \\ 1 & 3 & 3 \end{bmatrix}, B : \begin{bmatrix} -3 & -3 & 3 \\ -4 & -4 & 4 \\ -3 & -3 & 3 \end{bmatrix}$
- $C : \begin{bmatrix} -3 & -4 & -3 \\ -3 & -4 & -3 \\ 3 & 4 & 3 \end{bmatrix}, D : \begin{bmatrix} -1 & 0 & 1 \\ -3 & 0 & 3 \\ -3 & 0 & 3 \end{bmatrix}$

# Convolution Example 2

## Quiz

$$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} * \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

- $A: \begin{bmatrix} -1 & -3 & -3 \\ 0 & 0 & 0 \\ 1 & 3 & 3 \end{bmatrix}, B: \begin{bmatrix} -3 & -3 & 3 \\ -4 & -4 & 4 \\ -3 & -3 & 3 \end{bmatrix}$
- $C: \begin{bmatrix} -3 & -4 & -3 \\ -3 & -4 & -3 \\ 3 & 4 & 3 \end{bmatrix}, D: \begin{bmatrix} -1 & 0 & 1 \\ -3 & 0 & 3 \\ -3 & 0 & 3 \end{bmatrix}$

# SIFT

## Discussion

- Scale Invariant Feature Transform (SIFT) features are features that are invariant to changes in the location, scale, orientation, and lighting of the pixels.

# Histogram Binning Diagram

## Discussion

# HOG

## Discussion

- Histogram of Oriented Gradients features is similar to SIFT but does not use dominant orientations.

# Classification

## Discussion

- SIFT features are not often used in training classifiers and more often used to match the objects in multiple images.
- HOG features are usually computed for every cell in the image and used as features (in place of pixel intensities) in classification algorithms such as SVM.