CS540 Introduction to Artificial Intelligence

Lecture 18

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River Crossing Problem

Quiz

- Three married couples (husbands and wifes) need to cross the river. The boat requires at least one person to operate and holds no more than two people. No woman can be in the presence of another man unless her husband is also present (this is called the Jealous Husband Problem). What is the minimum number of times the boat needs to go across the river?

Summary

Discussion

- Search:

1. Uninformed: Breadth first search → Add states at the end → Remove states from the front → Complete + Optimal.
2. Uninformed: Depth first search → Add states to the front → Remove states to the front → Incomplete + Not optimal.
3. Uninformed: Itervative deepening search → DFS with depth limits $1, 2, \ldots$ → Complete + Optimal.
4. Informed: Uniform cost search.
5. Informed: Best first greedy search.
6. Informed: A search.
7. Informed: A star search.

Uniformed vs. Informed Search

Motivation

- Uninformed search means only the goal G and the successor functions s' are given.
- Informed search means which non-goal states are better is also known.

Heuristic Diagram

Motivation

Uniform Cost Search

Description

- Expand the vertices with the lowest current path cost $g(s)$ first.
- It is BFS with a priority queue based on $g(s)$.
- It is equivalent to BFS if $c = 1$ is constant on all edges.
- It is also called Dijkstra's Algorithm.

UCS Example 1

Quiz

- Given the following adjacency matrix. Find UCS expansion path.

–	S	A	B	C	D	E	G
S	$h = 6$	2	1	–	–	–	9
A	–	$h = 0$	–	2	3	–	–
B	–	–	$h = 6$	–	2	4	–
C	–	–	–	$h = 4$	–	–	4
D	–	–	–	–	$h = 1$	–	4
E	–	–	–	–	–	$h = 10$	–
G	–	–	–	–	–	–	$h = 0$

UCS Example 1 Diagram

Quiz

UCS Example 1 Expansion Path

Quiz

- In last year's lecture, multiple copies of the same state with different costs can be inserted into the frontier at the same time: that is not the standard practice.
- The same state with a lower cost should replace the copy in the frontier. If you refer to solutions or videos from last year, please note this change.

UCS Example 2

Quiz

- Find UCS expansion path

–	S	A	B	G
S	$h = 3$	1	999	–
A	–	$h = 1000$	1	–
B	–	–	$h = 1$	1
G	–	–	–	$h = 0$

- $A : S, A, B, G$
- $B : S, B, G$
- $C : S, B, A, G$
- $D : S, B, A, B, G$

UCS Example 2 Diagram

Quiz

Uniform Cost Search Performance

Discussion

- UCS is complete.
- UCS is optimal with any c.

Best First Greedy Search

Description

- Expand the vertices with the lowest heuristic cost $h(s)$ first.
- Use a priority queue based on $h(s)$.
- BFGS is not an abbreviation of Best First Greedy Search: BFGS is the Broyden Fletcher Goldfarb Shanno algorithm (a version of gradient descent).

Greedy Example 1

Quiz

- Given the following adjacency matrix. Find Greedy Search expansion path.

–	S	A	B	C	D	E	G
S	$h = 6$	2	1	–	–	–	9
A	–	$h = 0$	–	2	3	–	–
B	–	–	$h = 6$	–	2	4	–
C	–	–	–	$h = 4$	–	–	4
D	–	–	–	–	$h = 1$	–	4
E	–	–	–	–	–	$h = 10$	–
G	–	–	–	–	–	–	$h = 0$

Greedy Example 1 Diagram

Quiz

Greedy Example 2

Quiz

- Find Greedy expansion path

–	S	A	B	G
S	$h = 3$	1	999	–
A	–	$h = 1000$	1	–
B	–	–	$h = 1$	1
G	–	–	–	$h = 0$

- $A : S, A, B, G$
- $B : S, B, G$
- $C : S, B, A, G$
- $D : S, B, A, B, G$

Greedy Example 2 Diagram

Quiz

Best First Greedy Search Performance

Discussion

- Greedy is incomplete.
- Greedy is not optimal.

A Search

Description

- Expand the vertices with the lowest total cost $g(s) + h(s)$ first.
- Use a priority queue based on $g(s) + h(s)$.
- A stands for Always be optimistic?

A Search Example 1

Quiz

- Given the following adjacency matrix. Find A Search expansion path.

—	S	A	B	C	D	E	G
S	$h = 6$	2	1	—	—	—	9
A	—	$h = 0$	—	2	3	—	—
B	—	—	$h = 6$	—	2	4	—
C	—	—	—	$h = 4$	—	—	4
D	—	—	—	—	$h = 1$	—	4
E	—	—	—	—	—	$h = 10$	—
G	—	—	—	—	—	—	$h = 0$

A Search Example 1 Diagram

Quiz

A Search Example 2

Quiz

- Find A search expansion path

–	S	A	B	G
S	$h = 3$	1	999	–
A	–	$h = 1000$	1	–
B	–	–	$h = 1$	1
G	–	–	–	$h = 0$

- $A : S, A, B, G$
- $B : S, B, G$
- $C : S, B, A, G$
- $D : S, B, A, B, G$

A Search Example 2 Diagram

Quiz

A Search Performance

Discussion

- A is complete.
- A is not optimal.

A Star Search

Description

- $A^{\star}$ search is A search with an admissible heuristic.

Admissible Heuristic

Definition

- A heuristic is admissible if it never over estimates the true cost.

$$0 \leqslant h(s) \leqslant h^{\star}(s)$$

Admissible Heuristic 8 Puzzle Example

Definition

Admissible Heuristic 8 Puzzle Example

Quiz

- Which ones (select multiple) of the following are admissible heuristic function for the 8 Puzzle?
- $A : h(s) =$ number of tiles in the wrong position.
- $B : h(s) = 0$.
- $C : h(s) = 1$.
- $D : h(s) =$ sum of Manhattan distance between each tile and its goal location.
- $E : h(s) =$ sum of Euclidean distance between each tile and its goal location.

A Star Search Example 1

Quiz

- Given the following adjacency matrix. Find *A* Star Search expansion path.

–	S	A	B	C	D	E	G
S	$h = 6$	2	1	–	–	–	9
A	–	$h = 0$	–	2	3	–	–
B	–	–	$h = 6$	–	2	4	–
C	–	–	–	$h = 4$	–	–	4
D	–	–	–	–	$h = 1$	–	4
E	–	–	–	–	–	$h = 10$	–
G	–	–	–	–	–	–	$h = 0$

A Star Search Example 1 Diagram

Quiz

Admissible Heuristic General Example 1

Quiz

- Which ones (select multiple) of the following are admissible heuristic function?
- $A : h(s) = h^\star(s) \cdot 2$
- $B : h(s) = \sqrt{h^\star(s)}$
- $C : h(s) = h^\star(s) + 1$
- $D : h(s) = \min\{1, h^\star(s)\}$
- $E : h(s) = h^\star(s) \cdot \dfrac{1}{2}$
- $F : h(s) = h^\star(s)^2$
- $G : h(s) = \max\{1, h^\star(s)\}$
- $H : h(s) = h^\star(s) - 1$

A Search Performance

Discussion

- $A^\star$ is complete.
- $A^\star$ is optimal.

Iterative Deepening A Star Search

Discussion

- $A^{\star}$ can use a lot of memory.
- Do path checking without expanding any vertex with $g(s) + h(s) > 1$.
- Do path checking without expanding any vertex with $g(s) + h(s) > 2$.
- ...
- Do path checking without expanding any vertex with $g(s) + h(s) > d$.

Iterative Deepening A Star Search Performance

Discussion

- IDA* is complete.
- IDA* is optimal.
- IDA* is more costly than $A^\star$.

Beam Search

Discussion

- Version 1: Keep a priority queue with fixed size k. Only keep the top k vertices and discard the rest.
- Version 2: Only keep the vertices that are at most ε worse than the best vertex in the queue. ε is called the beam width.

Beam Search Performance

Discussion

- Beam is incomplete.
- Beam is not optimal.

Summary

Discussion

- Search:

1. Uninformed: Breadth first search.
2. Uninformed: Depth first search.
3. Uninformed: Itervative deepening search.
4. Informed: Uniform cost search → Remove states with lowest current cost (g) → Complete, Optimal.
5. Informed: Best first greedy search → Remove states with the lowest heuristic cost (h) → Incomplete, Not optimal.
6. Informed: A search → Remove states with the lowest current plus heuristic cost ($g + h$) → Complete, Not Optimal.
7. Informed: A star search → admissible heuristic (under-estimates true cost) → Complete, Optimal.