

CS540 Introduction to Artificial Intelligence

Lecture 22

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Based on lecture slides by Jerry Zhu, Yingyu Liang, and Charles Dyer

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Traveler's Dilemma

Quiz

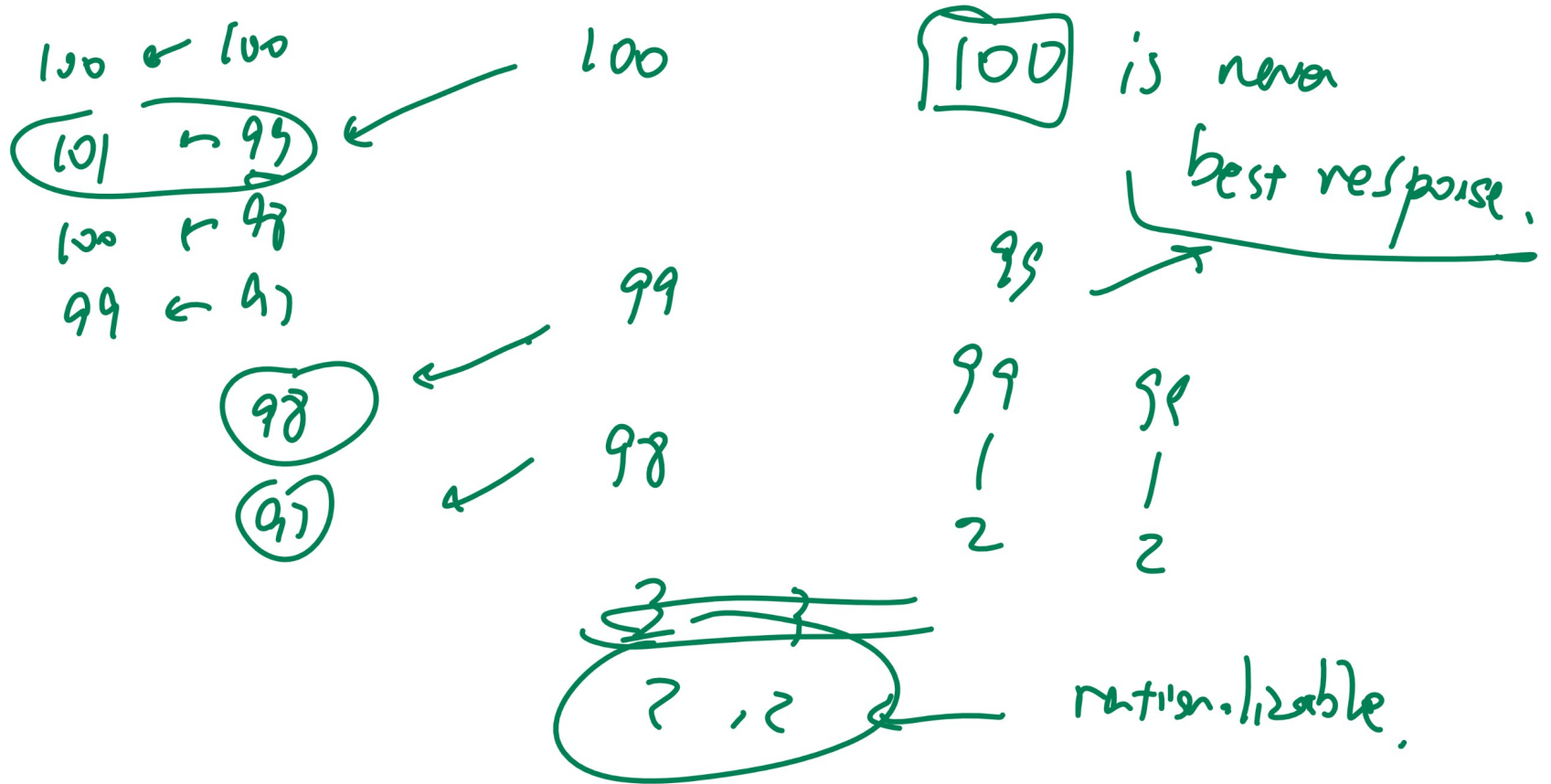
A1

- Two identical antiques are lost. The airline only knows that its value is at most 100 dollars, so the airline asks their owners (travelers) to report its value (non-negative integers, ≥ 2). The airline tells the travelers that they will be paid the minimum of the two reported values, and the traveler who reported a strictly lower value will receive 2 dollars in reward. If you are one of the travelers, what will you report?

(0, 0)

Traveler's Dilemma, Rationalizability

Quiz

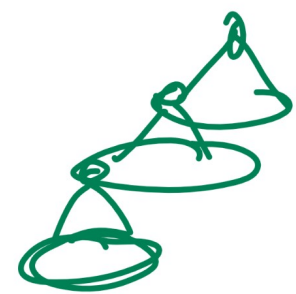


Summary

Discussion

- Adversarial Search:

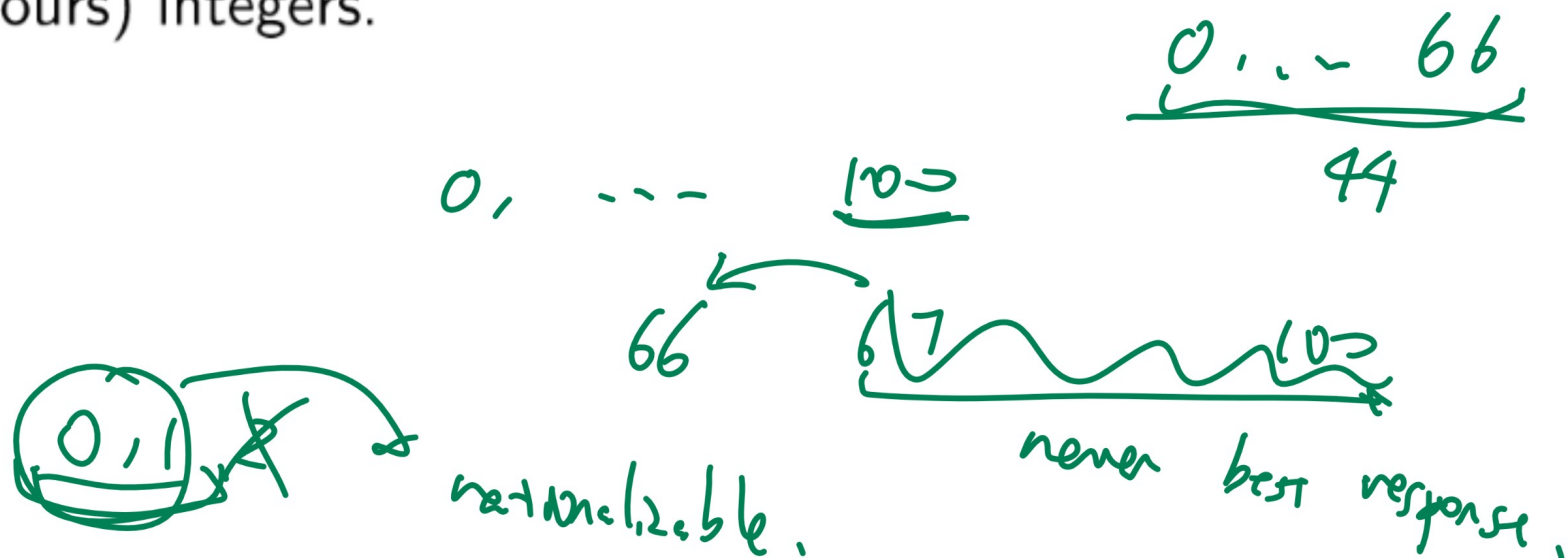
- Sequential Move Games: Minimax → DFS on the game tree.
- Sequential Move Games: Alpha-Beta Pruning → DFS to keep track α and β → prune the subtree with $\alpha \Rightarrow \beta$.
- Simultaneous Move Games: Iterated Elimination of Strictly Dominated Strategies (Rationalizability).
- Simultaneous Move Games: Nash Equilibrium.



Guess Average Game

Motivation

- Write down an integer between 0 and 100 that is the closest to two thirds ($2/3$) of the average of everyone's (including yours) integers.



Guess Average Game Derivation

Motivation

Rationalizability

Motivation

- An action is 1-rationalizable if it is the best response to some action.
- An action is 2-rationalizable if it is the best response to some 1-rationalizable action.
- An action is 3-rationalizable if it is the best response to some 2-rationalizable action.
- An action is rationalizable if it is ∞ -rationalizable.

Rationalizability Example

Quiz

- Both players are MAX players. Which actions are rationalizable for the ROW player?

Q2

—	A	B	C
A	(2, 4)	(3, 7)	(4, 5)
B	(1, 2)	(5, 4)	(2, 3)
C	(4, 1)	(2, 8)	(5, 3)
D	(3, 0)	(4, 0)	(1, 9)

Handwritten annotations on the table:

- Red wavy lines across each row, labeled "MAX" on the left.
- Blue wavy lines across each column, labeled "COL" at the top and bottom.
- Green circles around the first element of each payoff pair: (2), (3), (4), (1), (4), (2), (5), (3), (3), (4), (4), (1).
- Yellow circles around the second element of each payoff pair: (4), (7), (5), (2), (1), (8), (3), (9).
- A yellow arrow points from the (5, 4) cell to the word "rationalizable" written in blue and underlined.
- Green text at the bottom says "C is the best response to D".
- Green text at the bottom says "score / payoff / value of ROW".

Best Response

Definition

- An action is a best response if it is optimal for the player given the opponents' actions.

$$br_{MAX}(s_{MIN}) = \operatorname{argmax}_{s \in S_{MAX}} c(s, s_{MIN})$$

$$br_{MIN}(s_{MAX}) = \operatorname{argmin}_{s \in S_{MIN}} c(s_{MAX}, s)$$

Nash Equilibrium

Definition

- A Nash equilibrium is a state in which all actions are best responses.

Nash Equilibrium Example 1

Quiz

- Find the value of the Nash equilibrium of the following zero-sum game.

MIN ←

—	I	II	III
I	-4	-7	-3
II	9	1	7
III	-6	-1	5

MAX

NE

value of MAX

Nash Equilibrium Example 1

Quiz

- Find the value (of MAX player) of the Nash equilibrium of the following zero-sum game.

—	I	II	III
I	(-4, 4)	(-7, 7)	(-3, 3)
II	(9, -9)	(1, -1)	(7, -7)
III	(-6, 6)	(-1, 1)	(5, -5)

Handwritten annotations:

- Green arrow from "MAX" above pointing to column I.
- Red arrow from "MAX" on the left pointing to row II.
- Red box around the cell (II, II) containing (1, -1).
- Red circles around the values 7 in (I, II), 1 in (II, II), and 6 in (III, I).
- Red circles around the values -1 in (II, II) and 1 in (III, II).
- Red arrow from the bottom of the table pointing to the circled 1 in (III, II).
- Red underlined text "NE" below the table.
- Blue text "Sum up to 0" with a blue arrow pointing to the circled 1 in (III, II).
- Red "T" above the word "Nash" in the text above.

Nash Equilibrium Example 2

Quiz

- Find the value of the Nash equilibrium of the following zero-sum game.

MIN

—	I	II	III
I	1	2	3
II	4	5	6
III	7	8	9

MAX

- A: 1 , B: 3 , C: 5 , D: 7 , E: I don't understand

best response for col player

best response for row player

MIN
~~1, 2~~

Q3

Prisoner's Dilemma

Discussion

- A simultaneous move, non-zero-sum, and symmetric game is a prisoner's dilemma game if the Nash equilibrium state is strictly worse for both players than another state.

$x > 1$

$(x, x) > (1, 1)$
for both players.

	<u>Deny</u> C	<u>Confess</u> D
C	(x, x)	$(0, y)$
D	$(y, 0)$	$(1, 1)$

$y > x > 1$

→ NE

- C stands for Cooperate and D stands for Defect (not Confess and Deny). Both players are MAX players. The game is PD if $y > x > 1$. Here, (D, D) is the only Nash equilibrium and (C, C) is strictly better than (D, D) for both players.

Prisoner's Dilemma Derivation

Discussion

Properties of Nash Equilibrium

Discussion

	A	B
A	2	3
B	2	1

	A	B
A	2	3
B	1	1

- All Nash equilibria are rationalizable.
- No Nash equilibrium contains a strictly dominated action.
- Rationalizable actions (the set of Nash equilibria is a subset of this) can be found by iterated elimination of strictly dominated actions.
- The above statements are not true for weakly dominated actions.

Mixed Strategy Nash Equilibrium

Definition

- A mixed strategy is a strategy in which a player randomizes between multiple actions.
- A pure strategy is a strategy in which all actions are played with probabilities either 0 or 1.
- A mixed strategy Nash equilibrium is a Nash equilibrium for the game in which mixed strategies are allowed.

Rock Paper Scissors Example

Discussion

$$\frac{1}{3}R, \quad \frac{1}{3}P, \quad \frac{1}{3}S$$

- There are no pure strategy Nash equilibria.
- Playing each action (rock, paper, scissors) with equal probability is a mixed strategy Nash.

Rock Paper Scissors Example Derivation

Discussion

Battle of the Sexes Example

Quiz

- Battle of the Sexes (BoS, also called Bach or Stravinsky) is a game that models coordination in which two players have different preferences in which alternative to coordinate on.

	—	Bach	Stravinsky
Romeo		(x, y)	$(0, 0)$
Juliet		$(0, 0)$	(y, x)

$y > x > 0$

Battle of the Sexes Example 1

Quiz

- Find all Nash equilibria of the following game.

		I q	II $1-q$
P	I	(3, 5)	(0, 0)
1-P	II	(0, 0)	(5, 3)

Col
play I with
prob q
and II with
prob $1-q$

expect value I \geq ev 2

$$3q + 0(1-q) \geq 0q + 5(1-q)$$

$$q \leq \frac{5}{8}$$

$$q = \frac{5}{8} \quad q \geq \frac{5}{8}$$

BR_{row} \Rightarrow
 I
 I or II
 mix
 II

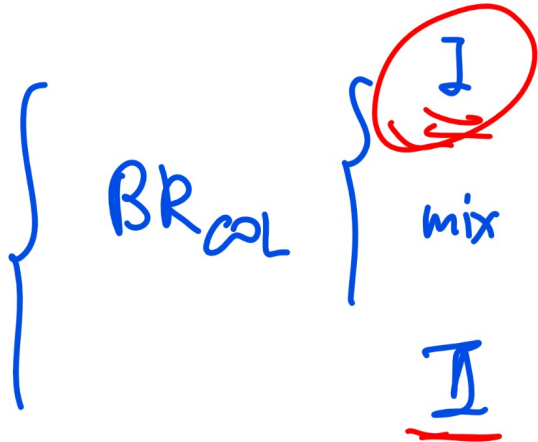
Battle of the Sexes Example 1 Derivation 1

Quiz

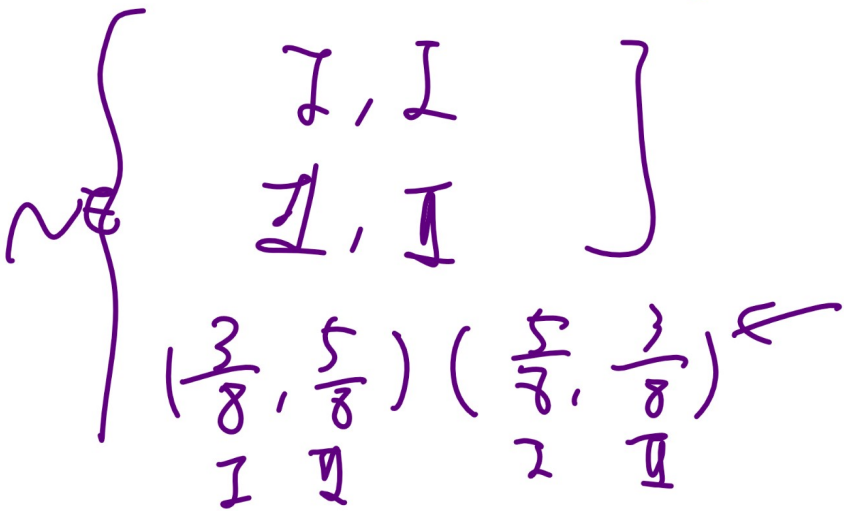
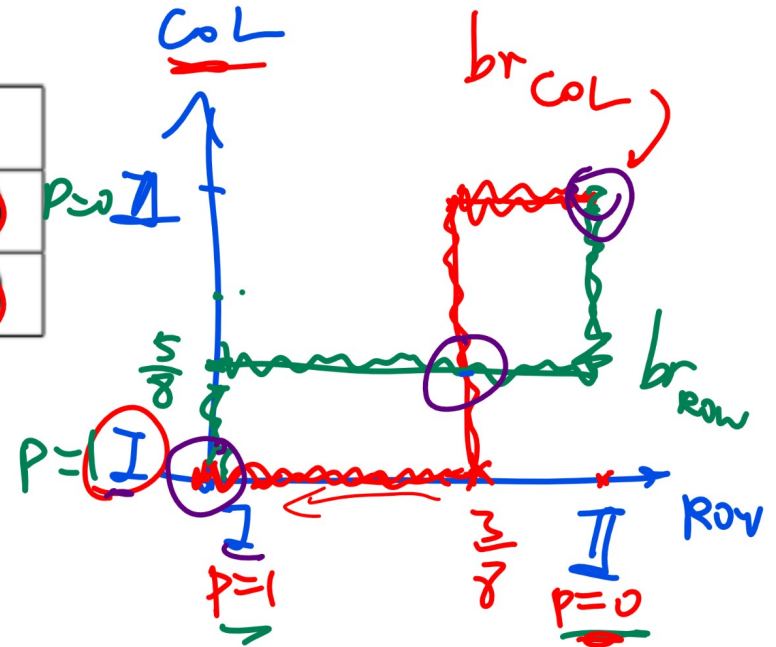
$$5p + 0(1-p) \geq 0p + 3(1-p) \Rightarrow p \geq \frac{3}{8}$$

$$p = \frac{3}{8}$$

$$p = \frac{3}{8}$$



	I	II
I	(3, 5)	(0, 0)
II	(0, 0)	(5, 3)



indifference condition
 $5p = 3(1-p)$

$$3p = 5(1-p)$$

Nash Theorem

Definition

- Every finite game has a Nash equilibrium.
- The Nash equilibria are fixed points of the best response functions.

Summary

Discussion

- Adversarial Search:
 - 1 Sequential Move Games: Minimax \rightarrow DFS on the game tree.
 - 2 Sequential Move Games: Alpha-Beta Pruning \rightarrow DFS to keep track α and $\beta \rightarrow$ prune the subtree with $\alpha \Rightarrow \beta$.
 - 3 Simultaneous Move Games: Iterated Elimination of Strictly Dominated Strategies (Rationalizability) \rightarrow Remove dominated actions for each player \rightarrow Repeat.
 - 4 Simultaneous Move Games: Nash Equilibrium \rightarrow Compute the best response \rightarrow Find strategies (pure or mixed) that are mutual best responses.