CS540 Introduction to Artificial Intelligence

Lecture 3

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Based on lecture slides by Jerry Zhu, Yingyu Liang, and Charles Dyer

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Two-thirds of the Average Game

Quiz

AND Operator Data

Quiz

Learning AND Operator

Quiz

OR Operator Data

Quiz

Learning OR Operator

Quiz

Learning XOR Operator

Quiz

Learning XOR Operator Network

Quiz

Single Layer Perceptron

Motivation

- Perceptrons can only learn linear decision boundaries.
- Many problems have non-linear boundaries.
- One solution is to connect perceptrons to form a network.

Decision Boundary Diagram

Motivation

Multi-Layer Perceptron

Motivation

- The output of a perceptron can be the input of another.

$$a = g\left(w^T x + b\right)$$

$$a' = g\left(w'^T a + b'\right)$$

$$a'' = g\left(w''^T a' + b''\right)$$

$$\hat{y} = \mathbb{1}_{\{a''>0\}}$$

Neural Network Biology

Motivation

- Human brain: $100,000,000,000$ neurons.
- Each neuron receives input from $1,000$ others.
- An impulse can either increase or decrease the possibility of nerve pulse firing.
- If sufficiently strong, a nerve pulse is generated.
- The pulse forms the input to other neurons.

Theory of Neural Network

Motivation

- In theory:

1. 1 Hidden-layer with enough hidden units can represent any continuous function of the inputs with arbitrary accuracy.
2. 2 Hidden-layer can represent discontinuous functions.

- In practice:

1. AlexNet: 8 layers.
2. GoogLeNet: 27 layers (or 22 + pooling).
3. ResNet: 152 layers.

Gradient Descent

Motivation

- The derivatives are more difficult to compute.
- The problem is no longer convex. A local minimum is no longer guaranteed to be a global minimum.
- Need to use chain rule between layers called backpropagation.

Backpropagation

Description

- Initialize random weights.
- (Feedforward Step) Evaluate the activation functions.
- (Backpropagation Step) Compute the gradient of the cost function with respect to each weight and bias using the chain rule.
- Update the weights and biases using gradient descent.
- Repeat until convergent.

Neural Network Demo

Motivation

Two-Layer Neural Network Weights Diagram 1

Motivation

Two-Layer Neural Network Weights Diagram 2

Motivation

Two-Layer Neural Network Weights Diagram 3

Motivation

Gradient Step, Combined

Definition

- Put everything back into the chain rule formula. (Please check for typos!)

$$\frac{\partial C}{\partial w_{j'j}^{(1)}} = \sum_{i=1}^{n} \left(a_i - y_i\right) a_i \left(1 - a_i\right) w_j^{(2)} a_{ij}^{(1)} \left(1 - a_{ij}^{(1)}\right) x_{ij'}$$

$$\frac{\partial C}{\partial b_j^{(1)}} = \sum_{i=1}^{n} \left(a_i - y_i\right) a_i \left(1 - a_i\right) w_j^{(2)} a_{ij}^{(1)} \left(1 - a_{ij}^{(1)}\right)$$

$$\frac{\partial C}{\partial w_j^{(2)}} = \sum_{i=1}^{n} \left(a_i - y_i\right) a_i \left(1 - a_i\right) a_{ij}^{(1)}$$

$$\frac{\partial C}{\partial b^{(2)}} = \sum_{i=1}^{n} \left(a_i - y_i\right) a_i \left(1 - a_i\right)$$

Gradient Descent Step

Definition

- The gradient descent step is the same as the one for logistic regression.

$$w_j^{(2)} \leftarrow w_j^{(2)} - \alpha \frac{\partial C}{\partial w_j^{(2)}}, j = 1, 2, ..., m^{(1)}$$

$$b^{(2)} \leftarrow b^{(2)} - \alpha \frac{\partial C}{\partial b^{(2)}},$$

$$w_{j'j}^{(1)} \leftarrow w_{j'j}^{(1)} - \alpha \frac{\partial C}{\partial w_{j'j}^{(1)}}, j' = 1, 2, ..., m, j = 1, 2, ..., m^{(1)}$$

$$b_j^{(1)} \leftarrow b_j^{(1)} - \alpha \frac{\partial C}{\partial b_j^{(1)}}, j = 1, 2, ..., m^{(1)}$$

Learning Logical Operators, XOR

Quiz

Learning Logical Operators, XOR, Diagram

Quiz

Learning Logical Operators, XOR, Answer

Quiz

Three-Layer Neural Network Weights Diagram

Motivation