Question 1

M6Q1

• [4 points] John tells his professor that he forgot to submit his homework assignment. From experience, the professor knows that students who finish their homework on time forget to turn it in with probability 0.87. She also knows that 0.4 of the students who have not finished their homework will tell her they forgot to turn it in. She thinks that 0.96 of the students in this class completed their homework on time. What is the probability that John is telling the truth (i.e. he finished it given that he forgot to submit it)?

not C

$\neg C$

$Pr\{\cancel{Forget} \mid \cancel{Finish}\} = 0.87$

F C

$Pr\{F \mid \neg C\} = 0.4$

$Pr\{C\} = 0.96$

$$Pr\{C \mid F\} = \frac{Pr\{\cancel{F \mid C}\}\cdot \cancel{Pr\{C\}}}{Pr\{F \mid C\}\cdot Pr\{C\} + Pr\{\cancel{F \mid \neg C}\}\cdot \cancel{Pr\{\neg C\}}}$$

0.87 0.96

0.4 0.04

$$= \frac{\boxed{Pr\{C, F\}}}{Pr\{F\}} = \frac{\boxed{Pr\{F \mid C\}\cdot Pr\{C\}}}{Pr\{F, C\} + Pr\{F, \neg C\}}$$

	F=0	F=1
C=0	$\neg F, \neg C$	$F, \neg C$
C=1	$\neg F, C$	F, C

F

$$\frac{\Pr\{F \mid C\} \cdot \Pr\{C\}}{\Pr\{F \mid C\} \cdot \Pr\{C\} + \Pr\{F \mid \neg C\} \cdot \Pr\{\neg C\}}$$

marginal of F.

$$\Pr\{F \mid C\} = \frac{\Pr\{F, C\}}{\Pr\{C\}}$$

$$\Pr\{F\} = \Pr\{F, C\} + \Pr\{F, \neg C\}$$

$$\boxed{\begin{aligned} &\Pr\{F, C, X\} + \Pr\{F, \neg C, X\} \\ &+ \Pr\{F, C, \neg X\} + \Pr\{F, \neg C, \neg X\} \end{aligned}}$$

Question 2

• [3 points] Assume the prior probability of having a female child (girl) is the same as having a male child (boy) and both are 0.5. The Smith family has 5 kids. One day you saw one of the Smith children, and she is a girl. The Wood family has 5 kids, too, and you heard that at least one of them is a girl. What is the chance that the Smith family has a boy? What is the chance that the Wood family has a boy?

Smith = 1 − prob all 4 kids are girls.

$$= 1 - \frac{1}{2^4}$$

G G G G
G G G B
G G B G
G G B B
G B G G
⋮

16

Wood = 1 − prob all 5 kids are girls.

$$= 1 - \frac{1}{2^5 - 1}$$

$\boxed{G\ G\ G\ G\ G}$
G G G G B
⋮

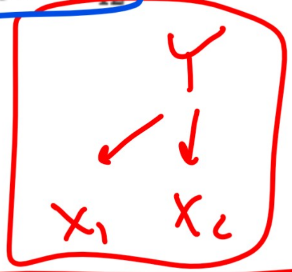
31

32

impossible → B B B B B

Question 1

• [4 points] Consider a classification problem with $n = 34$ classes $y \in \{1, 2, \ldots, n\}$, and two binary features $x_1, x_2 \in \{0, 1\}$. Suppose $\mathbb{P}\{Y = y\} = \frac{1}{34}$, $\mathbb{P}\{X_1 = 1 | Y = y\} = \frac{y}{50}$, $\mathbb{P}\{X_2 = 1 | Y = y\} = \frac{y}{42}$. Which class will naive Bayes classifier produce on a test item with $X_1 = 1$ and $X_2 = 1$.



Bayes net

$$\operatorname*{argmax}_y \Pr\{Y = y \mid X_1 = 1, X_2 = 1\}$$

$$\Downarrow$$

$$\Pr\{Y = y, X_1 = 1, X_2 = 1\} \;/\; \Pr\{X_1 = 1, X_2 = 1\}$$

indep irrelevant of y

$$\Downarrow$$

$$\underbrace{\Pr\{Y = y\}}_{\frac{1}{34}} \cdot \underbrace{\Pr\{X_1 = 1 \mid Y = y\}}_{\frac{y}{50}} \cdot \underbrace{\Pr\{X_2 = 1 \mid Y = y\}}_{\frac{y}{42}}$$

$$\rightarrow \operatorname*{argmax}_y \frac{y^2}{\text{constant}} = 34.$$

$y \to \infty$

$\rightarrow X_1 = 1, \; X_2 = 0$

$\Pr\{X_2 = 0 \mid Y = y\}$

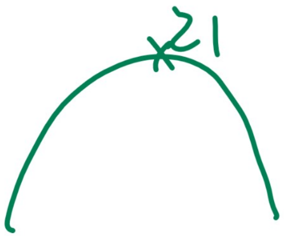
$$\operatorname*{argmax}_y \frac{1}{34} \boxed{\frac{y}{50}} \boxed{\left(1 - \frac{y}{42}\right)}$$

$1 - \Pr\{X_2 = 1 \mid Y = y\}$

$$\underbrace{y(42-y)}_{\text{constant}}$$

① try 1, 2 .. ~ 34 $\longrightarrow$ find max.

② treat $y \in [0, 34] \Longrightarrow \frac{\partial}{\partial y} y(42-y) \overset{\text{set}}{=} 0$



if 20.5 instead

$\rightarrow 42 - 2y = 0$

$y = \underline{21}$

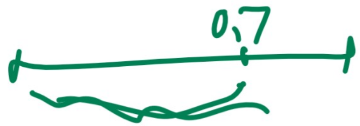
try 20, 21

P1 – P5

P6

Question 3

• [2 points] We have a biased coin with probability 0.94 of producing Heads. We create a predictor as follows: generate a random number uniformly distributed in (0, 1). If the random number is less than 0.7 we predict Heads, otherwise, we predict Tails. What is this predictor's (expected) accuracy in predicting the coin's outcome?



$$\Pr\{\underline{H}, H\} + \Pr\{\underline{T} . \underline{T}\}$$

$$= \Pr\{H\} \cdot \Pr\{H\} + \Pr\{T\} \cdot \Pr\{T\}$$

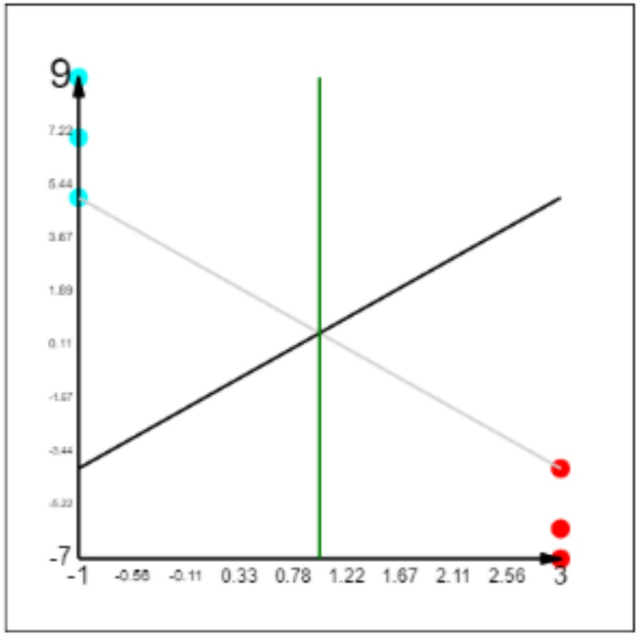
$$= 0.94 \cdot 0.7 + (1 - 0.94)(1 - 0.7) = \ldots$$

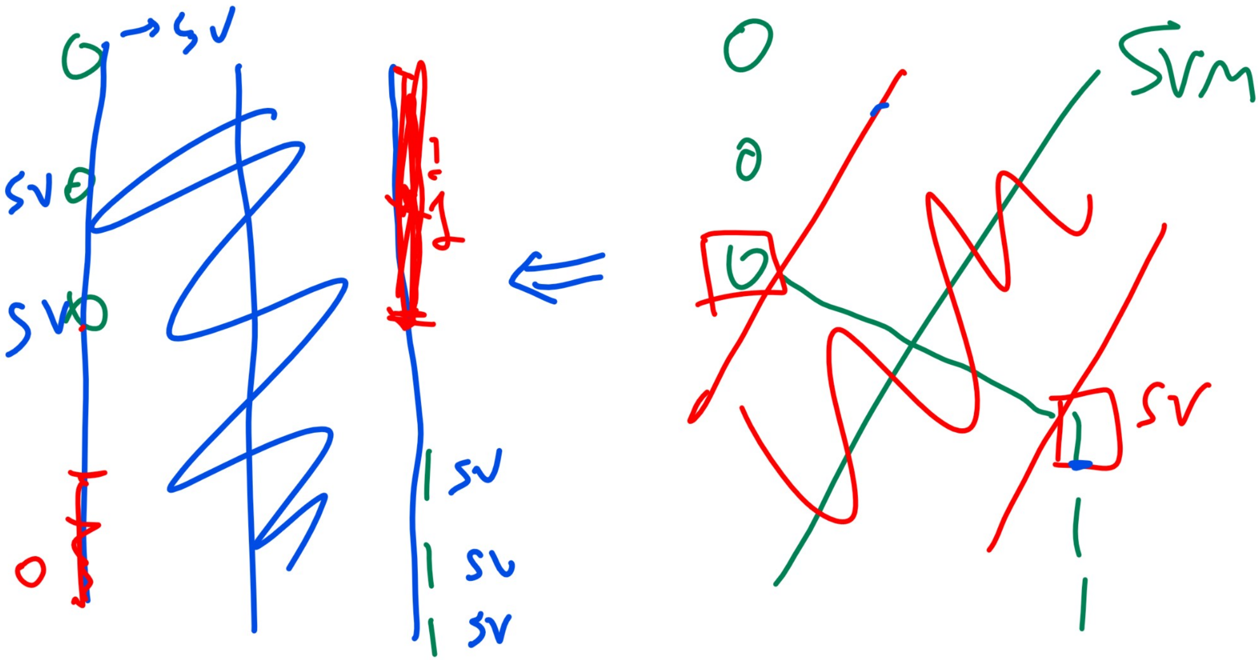
$$\Pr\{1 . 1\} + \Pr\{2, 2\} + \ldots \Pr\{6, 6\}$$

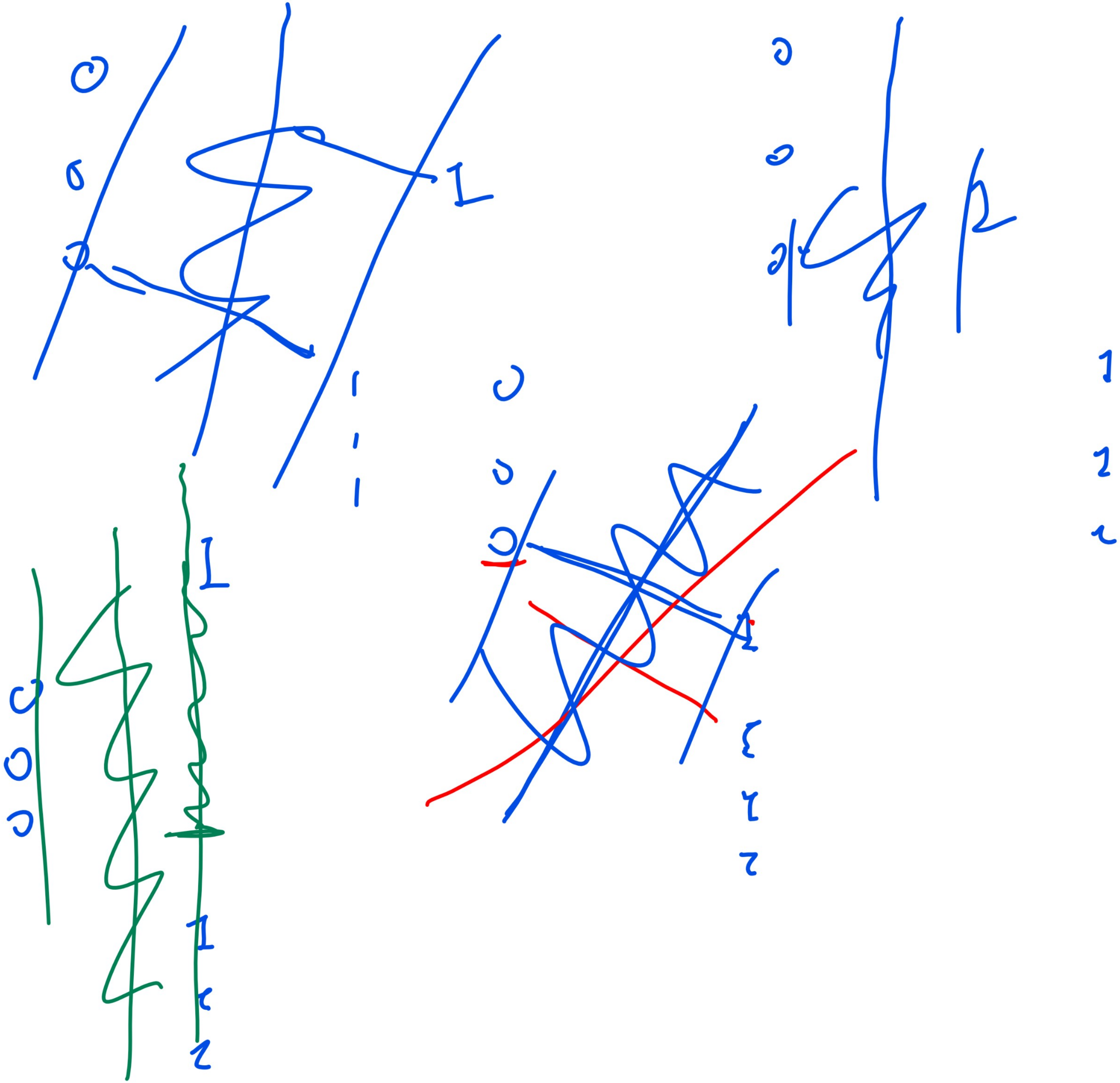
Question 6

- [4 points] Given the following training set, add one instance $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ with $y = 0$ so that all instances are support vectors for the Hard Margin SVM (Support Vector Machine) trained on the new training set.

x_1	x_2	y
3	-6	0
3	-7	0
3	-4	0
-1	5	1
-1	9	1
-1	7	1







Question 10

Mf Q10

• [2 points] You have a vocabulary with $\boxed{n = 885 \text{ word types}}$. You want to estimate the unigram probability p_w for each word type w in the vocabulary. In your corpus the total word token count $\sum_w c_w$ is 4621, and $c_{\text{dune}} = 10$.

Using Laplace smoothing (add 1), compute p_{dune}.

Crore

$\downarrow$

δ

$$\frac{\overbrace{c_{\text{dune}}}^{10} + 1}{\underbrace{\sum_w c_w}_{4621} + \underbrace{|\text{possible values of } w|}_{885}}$$

Question 4

M6 Q4

• [4 points] Some Na'vi's don't wear underwear, but they are too embarrassed to admit that. A surveyor wants to estimate that fraction and comes up with the following less-embarrassing scheme: Upon being asked "do you wear your underwear", a Na'vi would flip a fair coin outside the sight of the surveyor. If the coin ends up head, the Na'vi agrees to say "Yes"; otherwise the Na'vi agrees to answer the question truthfully. On a very large population, the surveyor hears the answer "Yes" for 0.94 fraction of the population. What is the estimated fraction of Na'vi's that don't wear underwear? Enter a fraction like 0.01 instead of a percentage 1%.

f

$$\Pr\{Yes\} = \Pr\{Yes, H\} + \Pr\{Yes, T\}$$

$$= \Pr\{Yes|H\}\cdot\Pr\{H\} + \Pr\{Yes|T\}\cdot\Pr\{T\}$$

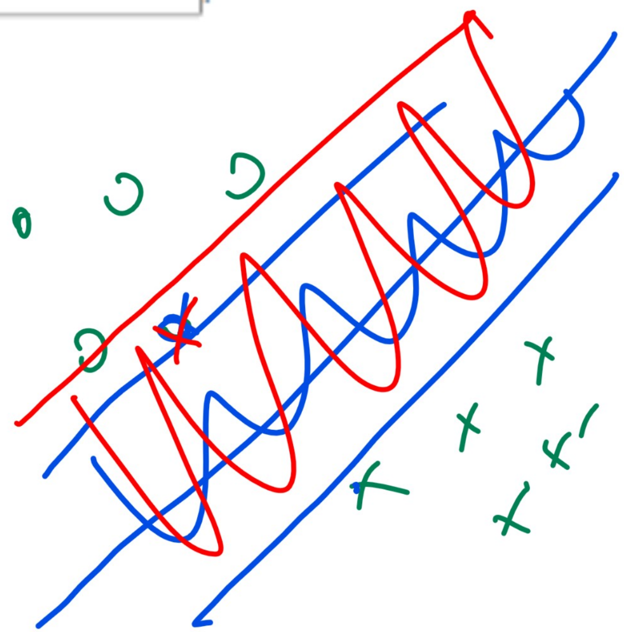
$$= 1\cdot 0.5 + \cancel{f}\,(1-f)\cdot 0.5$$

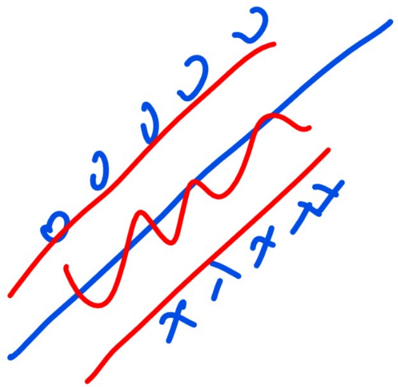
$$= 0.94 \quad\Longrightarrow\quad \text{solve for } f.$$

Question 8

X 1 Q8

- [4 points] Given a linear SVM (Support Vector Machine) that perfectly classifies a set of training data containing 10 positive examples and 7 negative examples. What is the minimum possible number of training examples that need be removed to cause the margin of a linear SVM to increase? If the answer is impossible, enter "-1".
- Answer: [1].





M6 Q6 → review session on Thursday.

ratioed.