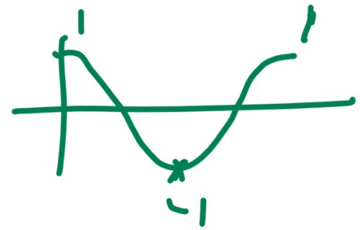
Question 7

M10

- [3 points] Consider a state space where the states are positive integers between 1 and 120. State i has two neighbors $i-1$ and $i+1$ (subject to the boundary constraints). State i has score $\boxed{\cos(i \cdot \pi) \cdot i^2}$. If one runs the hill climbing algorithm, which initial states can reach the global minimum? Break ties by moving towards the global minimum. If there are multiple global minima, list the states that lead to all of them.



▼ Hint

See Spring 2019 Midterm Q10, Fall 2017 Midterm Q8, Fall 2009 Midterm Q5. Try to solve the problem with a small n and find the pattern. Alternatively, start with the global minimum $i^\star$, keep count the states while $s(i-1) \geq s(i), i < i^\star$ or $s(i+1) \geq s(i), i > i^\star$.

- Answer (comma separated vector): 118, 119, 120

$\cos(i\pi)$

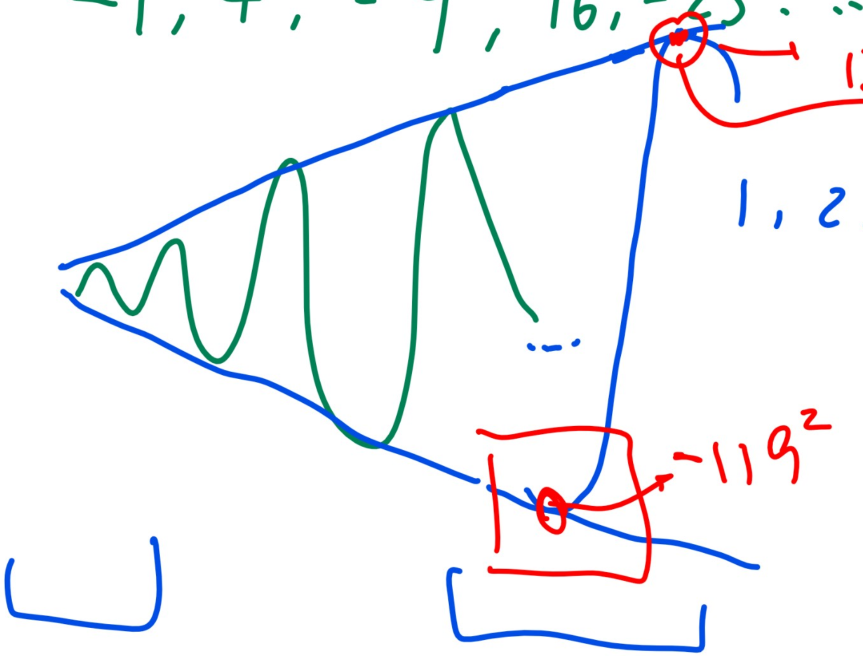
$-1, 4, -9, 16, -25, \ldots$

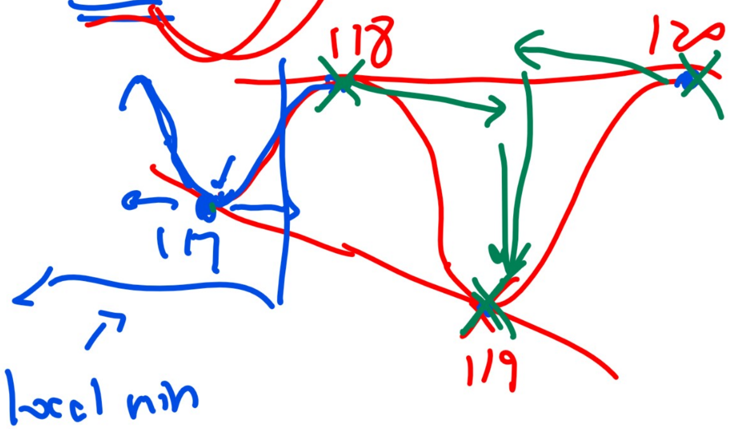
$(-1)^i \, i^2$

i^2

$1, 4, 9, \ldots$

$1, 2, \ldots, 120$





Question 10

M10

- [3 points] Four individuals (i.e. candidate solutions) in the current generation are given by 6-digit (6 dimensional) sequences: $[d_1 \quad d_2 \quad d_3 \quad d_4 \quad d_5 \quad d_6]$. Individual 1: $[8 \quad 5 \quad 3 \quad 7 \quad 9 \quad 5]$; Individual 2: $[7 \quad 4 \quad 5 \quad 9 \quad 5 \quad 5]$; Individual 3: $[9 \quad 1 \quad 9 \quad 5 \quad 9 \quad 6]$; Individual 4: $[8 \quad 2 \quad 8 \quad 1 \quad 7 \quad 7]$. The fitness function is $-d_1 - d_6$. What is the result of performing 1-point crossover for the sequences with the highest fitness (break ties by preferring the sequence that appears earlier in the list) with a cross-point between digit 4 and digit 5.
- Note: the first line representing the first child should start with the sequence with the highest fitness, and the second line representing the second child should start with the sequence with the second highest fitness.

▼ Hint

See Spring 2017 Midterm Q2. Compute the fitness of all four sequences and find the two with the highest fitness say $d^{(1)}, d^{(2)}$. The crossover of the two sequences between digits i and $i+1$ are $d_1^{(1)}, \ldots, d_i^{(1)}, d_{i+1}^{(2)}, \ldots, d_6^{(2)}$ and $d_1^{(2)}, \ldots, d_i^{(2)}, d_{i+1}^{(1)}, \ldots, d_6^{(1)}$.

- Calculator: Calculate .

~~loss/cost~~

score/fitness

- Answer (matrix with 2 lines, each line is a comma separated vector): ~~9,1,9,5,9,6~~ ~~8,5,3,7,9,5~~ .

1 -13

2 -12 ← highest

3 -15

4 -15

digit 4 | digit 5

parent 1: 8 5 3 7 | 9 5

parent 2: 7 4 5 9 | 5 5

child 2: 8 5 3 7 5 5

child 1: 7 4 5 9 9 5

Question 2

M10

- [3 points] Let h_1 be an admissible heuristic from a state to the optimal goal, A* search with which ones of the following h will never be admissible?

▶ Hint

- Choices:

- [ ] $\mathrm{h}(n) = \min(\mathrm{h}_1(n), 0)$
- [x] $\mathrm{h}(n) = \mathrm{h}_1(n) + 1$
- [ ] $\mathrm{h}(n) = \frac{\mathrm{h}_1(n)}{4}$
- [x] $\mathrm{h}(n) = \mathrm{h}_1(n) - 1$
- [x] $\mathrm{h}(n) = e^{\mathrm{h}_1(n)}$
- [ ] None of the above

$$h(goal) = \underbrace{h_1(goal)}_{0} - 1 = -1 \leq 0$$

$$h_1 = \frac{1}{100} h^*$$

$$\underbrace{h(n)}_{0} = 100 \underbrace{h_1(n)}_{0} << h^*(n)$$

$$0 \leq h_1(goal) = 0 \leq \underbrace{h^*(goal)}_{0}$$

$$h_1(goal) = 0$$

$$h(n) = h_1(n)^3$$

$$h_1 = h^{*\frac{1}{3}}$$

not never adm.

Question 3

- [3 points] If h_1 and h_2 are both admissible heuristic functions, which ones of following are also admissible heuristic functions?

always

▼ Hint

See Fall 2019 Midterm Q4 Q7 Q8 Q9, Fall 2018 Midterm Q5, Spring 2018 Midterm Q2, Fall 2006 Final Q3, Fall 2006 Midterm Q7, Fall 2005 Final Q3, Fall 2005 Midterm Q2. Since h_1 and h_2 are admissible, for any n, $0 \leq h_1(n) \leq h^\star(n)$ and $0 \leq h_2(n) \leq h^\star(n)$. It means if $0 \leq h_3(n) \leq h_1(n)$ or $0 \leq h_3(n) \leq h_2(n)$, then it must be true that $0 \leq h_3(n) \leq h^\star(n)$, i.e. h_3 is also admissible.

- Choices:

- [ ] $h_3(n) = h_1(n) + h_2(n)$?
- [x] $h_3(n) = \max(h_1(n), h_2(n))$
- [x] $h_3(n) = h_2(n)$ ✓
- [ ] $h_3(n) = \sqrt{|h_1(n) - h_2(n)|}$?
- [x] $h_3(n) = \frac{1}{2} h_1(n) + \frac{1}{2} h_2(n)$
- [ ] None of the above

≥ 0

check $\leq h^*$

$\boxed{\text{sub } h_1 = h_2 = h^*}$

$h_3 = h^* + h^* = 2h^* > h^*$

$h_1 = h_2 = 1 = h^* \quad h_3 = 2 > 1$

$h_3 = \max(h^*, h^*) = h^* \leq h^*$

$h_3 = \sqrt{h^* - h^*} = 0 \leq h^*$?

trick try 0.5

$h_1 = 0.25, \quad h_2 = 0, \quad h^* = 0.25$

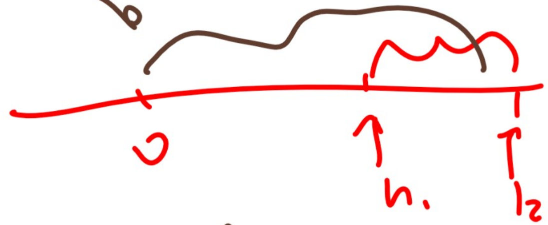
$h_3 = \sqrt{|0.25 - 0|} = \sqrt{0.25} = 0.5 > 0.25$

$h_3 \leq \frac{1}{2} h^* + \frac{1}{2} h^* = h^*$

$h_3 = \sqrt{|h_2 - h_1|}$ if h_2, h_1 are integers.

$\le |h_2 - h_1|$ $\sqrt{x} \le x$

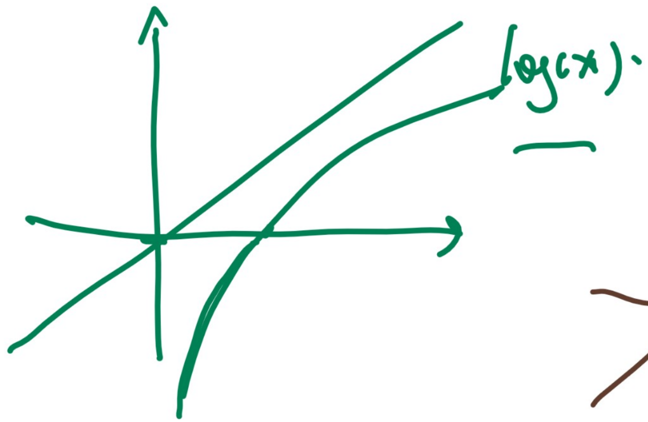
$\le \max\{|h_2|, |h_1|\}$ $h_2 \ge 0, h_1 \ge 0$

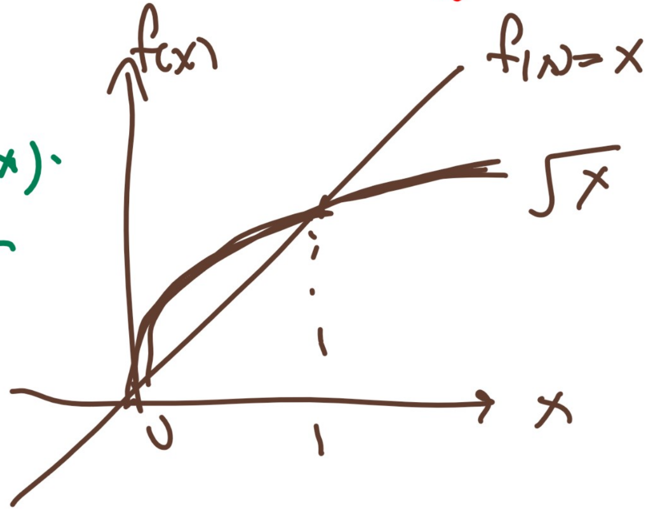


$\le \max\{h_2, h_1\}$

$\le \max\{h^*, h^*\}$

$= h^*$





Question 4

n 9

• [2 points] Consider n + 1 = 29 + 1 states. The initial state is 1, the goal state is n. State 0 is a dead-end state with no successors. For each non-0 state i, it has two successors: i + 1 and 0. There is no cycle check nor CLOSED list (this means we may expand (or goal-check) the same nodes many times, because we do not keep track of which nodes are checked previously). How many goal-checks will be performed by Breadth First Search? Break ties by expanding the node with the smaller index first.

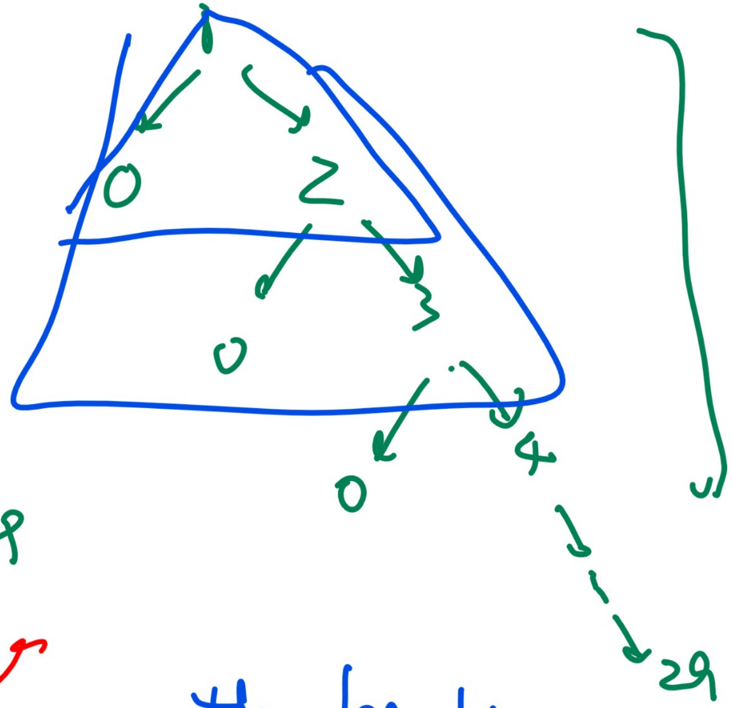
▶ Hint

• Answer: [ ] Calculate

BFS ~~1~~ ~~0~~ ~~2~~ ~~0~~ ~~3~~ ~~0~~ ~~4~~ ......

28 + 29

zeros 1 to 29 state



DFS ~~1~~ ~~0~~ ~~2~~ 0 3 ..

IDS 1 | 1 0 2 | 1 0 2 0 3 ....

level 0 level 1 level 2

0·2+1 1·2+1 2·2+1 28·2+1

1 + 3 + 5 + ... + 57

tie breaking large index first

1, 2, 3 .. 29

$$\sim 57 + 55 + 53 + \ldots + 1$$

$$\underbrace{58 \quad 58 \quad 58 \quad \ldots \quad 58}_{\cancel{28}\ 29}$$

$$58 \cdot 29 / 2 = \sim$$

Question 5

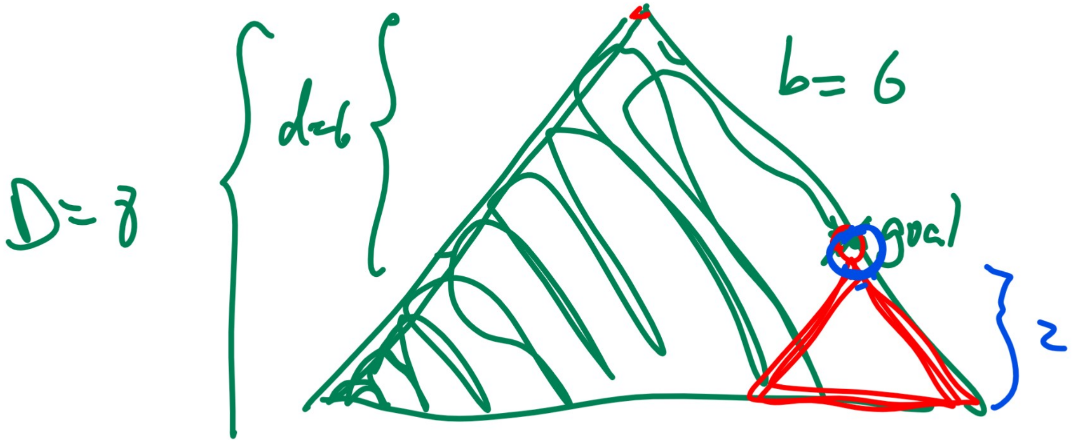
• [2 points] Consider a search tree where the root is at depth 0, each internal node has 6 children, and all leaves are at depth 8. There is a single goal state at depth 6. How much stack space (in number of states including the root and the goal) is sufficient so DFS always succeeds? Select all that applies.

▶ Hint

• Choices:

- [ ] 22
- [ ] 38
- [ ] 8
- [ ] 60
- [ ] 35
- [ ] None of the above

• Calculator: [   ] Calculate



total # nodes, $1 + 6 + 6^2 + 6^3 + \ldots 6^8$

nodes in the red subtree : $\cancel{1} + 6 + 6^2$

$1 + 6^3 + 6^4 + 6^5 + 6^6 + 6^7 + 6^8$

Question 10

• [2 points] Recall in uniform-cost search, each node has a path-cost from the initial node (sum of edge costs along the path), and the search expands the least path-cost node first. Consider a search graph with $n = 30$ nodes: $1, 2, \ldots, n$. For all $1 \le i < j \le n$, there is a directed edge from i to j with an edge cost 1. The initial node is 1, and the goal node is n. How many (unique) goal-checks (the same nodes expanded twice is counted only once) with uniform-cost search perform? Break ties by expanding the node with the $\boxed{\text{smaller}}$ index first.

$\boxed{\Sigma}$

larger

3 4 5 6 30

4 5 6

~~1~~ ~~2~~ ~~3~~ ~~4~~ ~~5~~ ~~6~~ ... ~~30~~

g 0 1 1 1 1 1 1

h

1, 30,

Smaller

30