Question 1

• [3 points] Given three clusters, $A = \{[-10], [-8]\}$, $B = \{x\}$, $C = \{[6], [6]\}$. Find a value of x so that A and B will be merged in the next iteration of single linkage hierarchical clustering, and B and C will be merged in the next iteration of complete linkage hierarchical clustering. Break ties by merging with the cluster with the smaller index (i.e. A, then B, then C).

• Note: there can be multiple answers, including non-integer answers, enter one of them. If there are none, enter 0.

• Answer: [ ] Calculate

$$\begin{cases} x + 8 < 6 - x \\ x + 10 > 6 - x \end{cases} \qquad \begin{cases} 2x < -2 \\ 2x > -4 \end{cases}$$

$$-2 \leq x \leq -1$$

$$x = -1.5$$

Question 2

• [4 points] Given the list of states in the priority queue (frontier) and the current cost g and heuristic cost h, what is the largest value of x so that state 0 will be removed (expanded) from the priority queue next in all three informed search strategies: UCS (Uniform Cost Search), (Best First) Greedy Search, and A Search? Break ties by expanding the state with the smallest index.

State	0	1	2	3	4	5
g	$\boxed{3}$	6	5	7	8	8
h	$\boxed{x}$	10	4	9	8	10
		16	9	16	16	18

• Answer: 4 Calculate

UCS ✓

GS

$x \le 4$

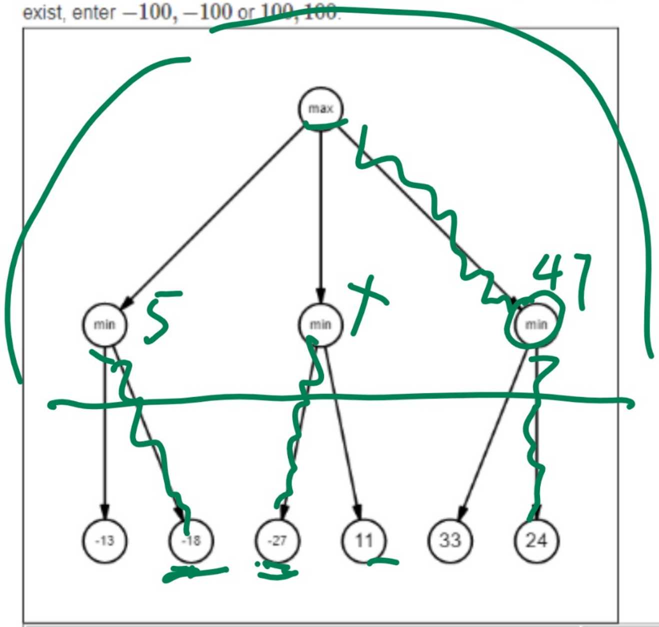
A

$3 + x \le 9$

$x \le 6$

Question 3

- [4 points] For a zero-sum game in which MAX moves first and the value to the MAX player is given in the diagram below, consider the static board evaluation (heuristic function) at the internal states provided in the table below. What are the smallest and largest possible values of x above and below which IDS (iterative deepening search) with depth limit 1 will find the correct solution for the game? You can assume all values are between -100 and 100. Enter two numbers between -100 and 100 (possibly including $-100, 100$).
- Note: for example, if you think $10 < x < 20$, enter $10, 20$; if you think any $x > 10$ works, enter $10, 100$; if you think any $x < 20$ works, enter $-100, 20$; if you think every x is okay, enter $-100, 100$; if you think no such x exist, enter $-100, -100$ or $100, 100$.



guess

IDS

State (Action)	Left	Middle	Right
Static Board Evaluation	5	x	47

$47 > 5$

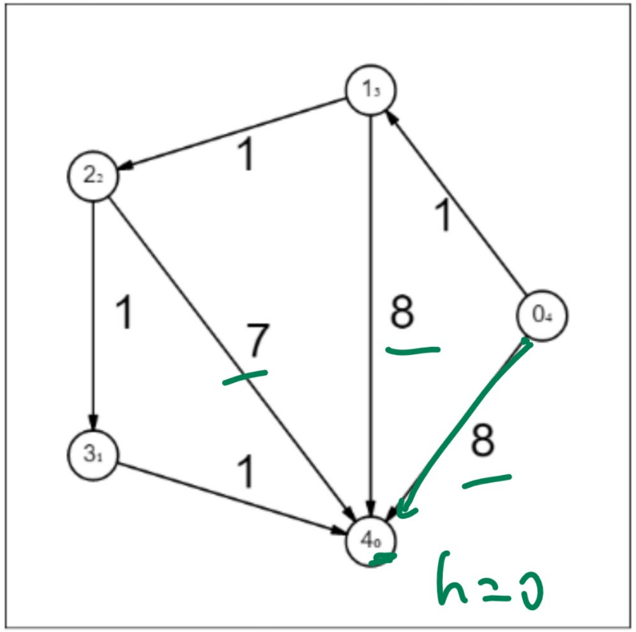
$\boxed{47 > x}$

no need $x < 5$

$[-100, 47)$

Question 8

• [4 points] Run Greedy (Best First Greedy Search) search algorithm on the following graph, starting from state 0 with the goal state being 4. Write down the expansion path (in the order of the states expanded). The heuristic function h is shown as subscripts. Break tie by expanding the state with a smaller index.



GS: ~~0~~ 1, ~~4~~ → 0, 4

h 3 0

UCS: ~~0~~ ~~1~~ ~~4~~ ~~2~~ ~~3~~

A 1 ~~8~~ 2 3

4

→ 0, 1, 2, 3, 4,

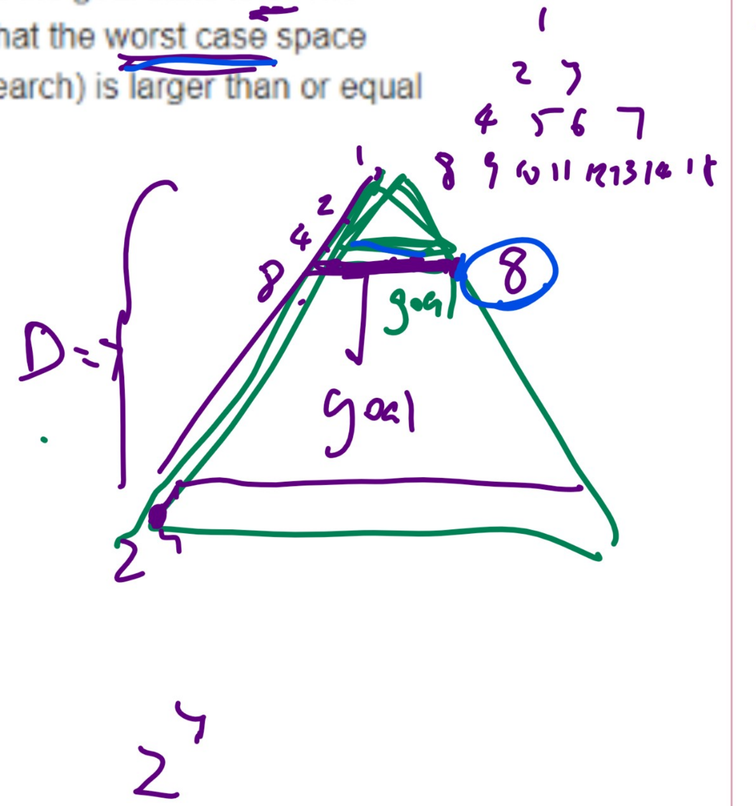
• In case the diagram is not clear: the weights are (with heuristic values on the diagonal entries):

$$\begin{bmatrix} 4 & 1 & 0 & 0 & 8 \\ 0 & 3 & 1 & 0 & 8 \\ 0 & 0 & 2 & 1 & 7 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

• Answer (comma separated vector): ______.

Question 1

- [3 points] Suppose the states are integers between 1 and x. The initial state is 1, and the goal state is 8. The successors of a state i are $2i$ and $2i + 1$, if exist. What is the smallest value of x so that the worst case space complexity (number of states stored in the list (queue or stack)) of DFS (Depth First Search) is larger than or equal to BFS (Breadth First Search)?
- Note: the worst case space complexity for BFS is b^d and for DFS is $(b - 1)D + 1$.
- Answer: 2^7 . Calculate



$$(b-1)D+1 \geq b^d$$

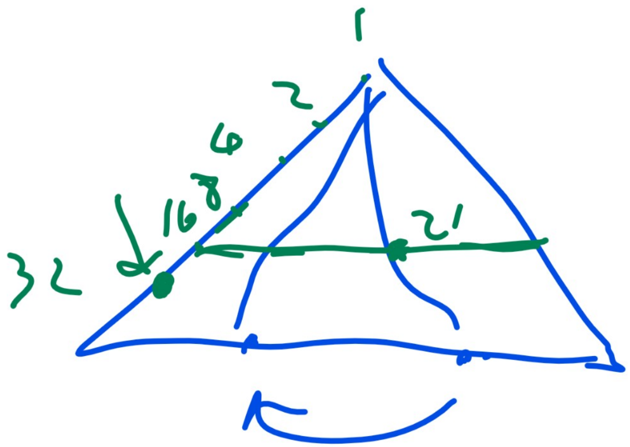
$$1D + 1 \geq 2^3 = 8$$

$$D \geq 7$$

$$2^7$$

Question 2

- [4 points] Suppose the states are integers between 1 and 256. The initial state is 1, and there are two goal states, the optimal one: 21, and another one with (strictly) higher cost (length of path from initial state to goal state): x. The successors of a state i are $2i$ and $2i + 1$, if exist. What is the smallest value of x such that DFS (Depth First Search) does not find the optimal goal in the worst case?
- Answer: 32 . Calculate



Question 3

- [4 points] The Nash equilibrium of the following simulatneous move zero-sum game is (U, L): the entry marked by x. What is the smallest and largest possible values of x? Enter two numbers.
- Note: if there is only one possible value, enter the same value twice; and if no values are possible, enter $0, 0$.

MAX \ MIN	L	C	R
U	x	10	3
M	1	5	3
R	-5	-2	7

- Answer (comma separated vector): 1, 3 Calculate

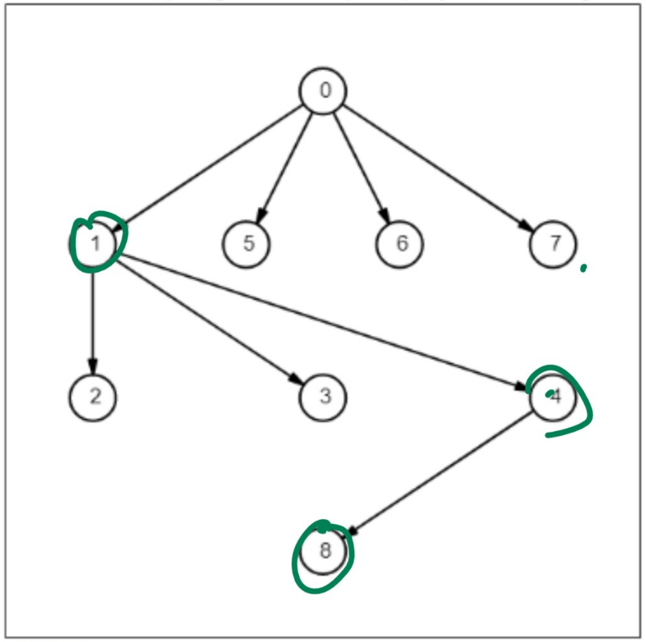
$x \geq 1$
$x \geq -5$
U is the br_{row}

$x \leq 10$
$x \leq 3$
L is the br_{col}

$1 \leq x \leq 3$

Question 11

• [4 points] Which order of goal check is possible with DFS (Depth First Search), without specifying the order of successors when putting them in the queue (i.e. you can rearrange the order of the branches)?



impossible 8 before 1, 4

2, 3, 4 before 1

anything before 0

• Choices:

(1) 3 before 2

(2) 6 before 5

(3) 8 before 5

(4) 7 before 3

(5) 8 before 7

(6) None of the above

• Answer (comma separated vector):

Question 1

• [3 points] The initial state and goal state of an 8-puzzle is given below. If the heuristic is the sum of Manhattan distances between the current position of each tile and the goal position, what is the heuristic of the initial state?

The goal state is:

1	2	3
4	5	6
7	8	0

The initial state is:

1	2	4
3	5	6
7	8	0

• Answer (comma separated vector): ______ Calculate

$$\left.\begin{array}{l} d(3,4) + \\ d(4,3) \end{array}\right\} \quad 3+3=6$$

Question 2

• [3 points] There are two players and k coins on the table. Players move sequentially with player 1 moving first. Each player chooses to take either one or two coins from the table. The player who takes the last coin wins. For which of the following values of $k = \{21; \ 18; \ 25; \ 28; \ 26; \ 32\}$ does the first player has a winning strategy? Enter the values of k, not the indices.

• Answer (comma separated vector): 25, 28 Calculate

26, 32

Take 1

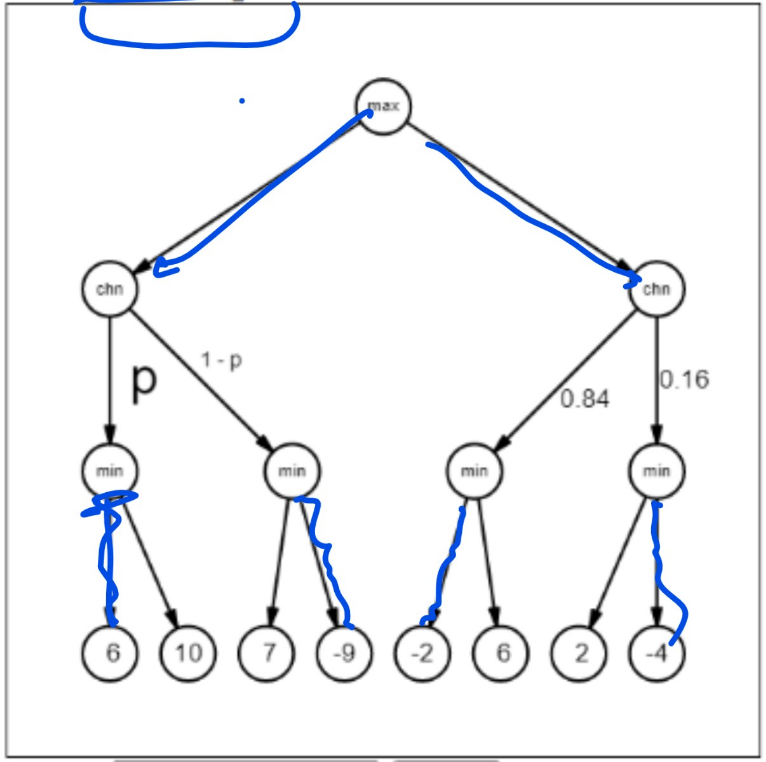
take 2

$\frac{10}{}$

$\frac{1}{}$

Question 3

• [4 points] For a zero-sum game in which MAX moves first and if the action Left is chosen, then Chance (Chn) moves Left with probability p and Right with probability $1-p$. Suppose the player who moves first uses a mixed strategy $\frac{1}{2}$ Left and $\frac{1}{2}$ Right in a solution, what is the value of p? If it's impossible, enter -1.



• Answer: Calculate

$6 \cdot p + -9(1-p)$

$= -2 \cdot 0.84 + -4 \cdot 0.16$

Solve p

$=$

{ if $p < 0$, $p > 1$ enter -1.

Question 4

• [4 points] Suppose the score (fitness) of a state (d_1, d_2, d_3, d_4) is $d_1 + d_2 + d_3 + d_4$, and only 1-point crossover with the cross-over point between d_2 and d_3 is used in a genetic algorithm (i.e. mutation probabilities are 0). Two states are chosen as parents at random according to the reproduction probabilities, what is the probability that one of their children is the optimal state (i.e. $(1, 1, 1, 1)$? Enter a number between 0 and 1.

• Note: a the two parents are sampled without replacement, meaning the probability that two states are chosen as parents is the product of their reproduction probabilities.

$1/9$

Index	1	2	3	4
State	[0 1 1 1]	[0 0 0 1]	[1 0 0 1]	[1 1 0 1]
fit	3	1	2	3
p	$\frac{3}{9}$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$

$\frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$

child 1 0 1 0 1

child 2 1 1 1 1

Question 5

• [3 points] Given the variance matrix $\hat{\Sigma} = \begin{bmatrix} 0 & \frac{1}{\sqrt{2}} & \boxed{\frac{-1}{\sqrt{2}}} \\ 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$, what is the first principal component? Enter a unit vector.

• Answer (comma separated vector): $-1/\sqrt{2}, 0, 1/\sqrt{2}$ Calculate

Question 6

• [3 points] There are $n = 43$ cookies. The brother first propose a division of these cookies into two piles (two integers adding up to n) and then the sister take one of the two piles. Both the brother and the sister want to maximize the number of cookie they take. What is the value of the game to the brother (measured by the number of cookies he gets)? Enter an integer.

• Answer: 21 Calculate

2 piles

21, 22 $\rightarrow$ 22, 21