Design of Search by Committees

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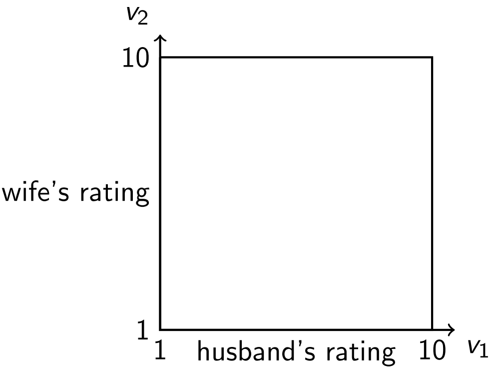
Story 1

Couple's House

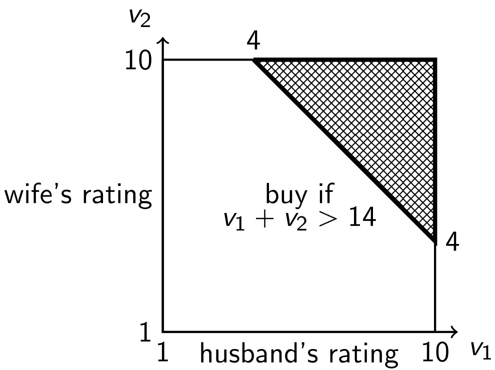
- A married couple looks for houses until they decide on purchasing one.
- The Canadian housing market is competitive and a house is gone before a new one becomes available.



Story 1

Couple's House, Example: Sum Rule

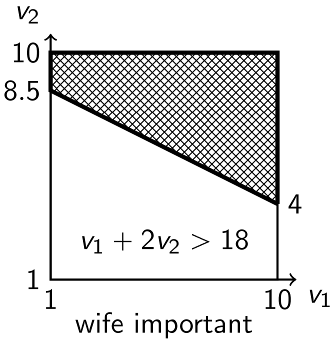
- A married couple looks for houses until they decide on purchasing one.
- The Canadian housing market is competitive and a house is gone before a new one becomes available.



Story 1

Couple's House, Example: Weighted Rules

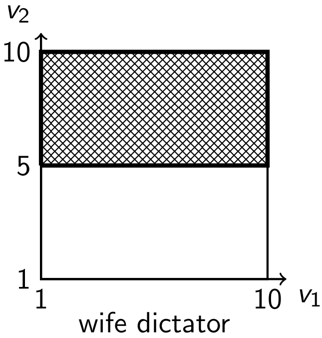
- A married couple looks for houses until they decide on purchasing one.
- The Canadian housing market is competitive and a house is gone before a new one becomes available.

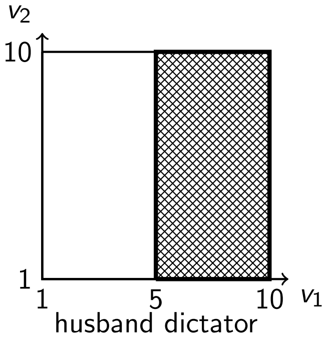


Story 1

Couple's House, Example: Dictator Rules

- A married couple looks for houses until they decide on purchasing one.
- The Canadian housing market is competitive and a house is gone before a new one becomes available.

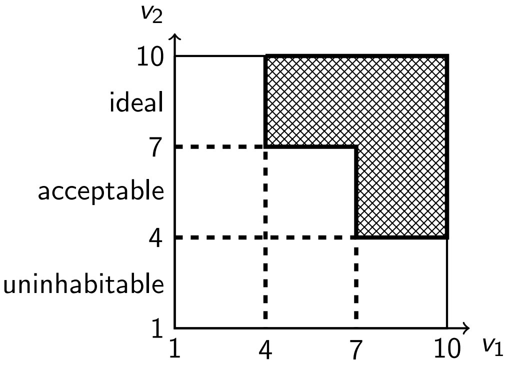




Story 1

Couple's House, Example: Ternary Rule

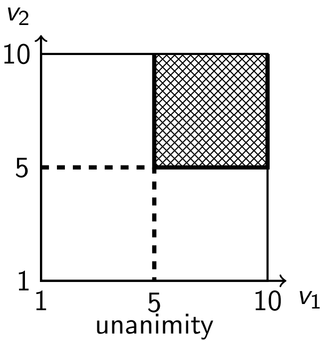
- A married couple looks for houses until they decide on purchasing one.
- The Canadian housing market is competitive and a house is gone before a new one becomes available.



Story 2

Committee Search, Example: Unanimity Rule

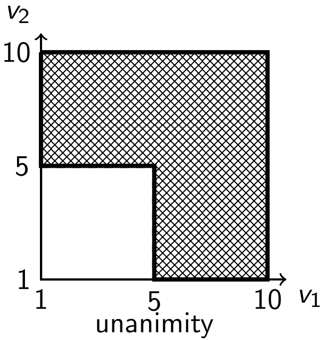
- A hiring committee receives job applications and conducts interviews until a position is filled.
- A decision is made right after each interview and is irreversible in small schools.



Story 3

Co-authors' Research, Example: Reverse Unanimity Rule

- Co-authors gain access to new data sets periodically.
- The authors have differing opinions on whether a data set can lead to interesting results.



Problem

- Committee search problems that have

1. sequential decision,
2. irreversible decision,
3. private value,
4. public allocation, and
5. no transfers.

Question

- Agents face stopping problem.
- Principal implements stopping rules.
- Which decision rules are implementable (incentive compatible)?

Answer

Short Version

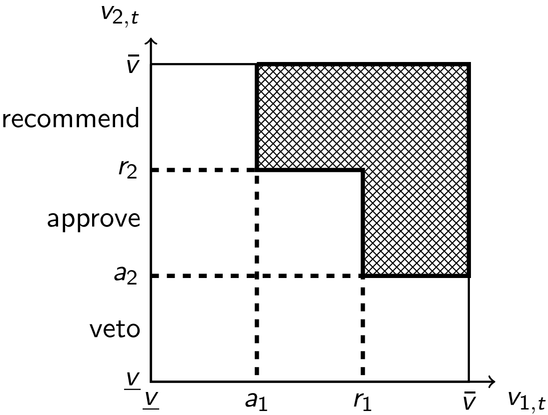
- Which allocation rules are implementable (incentive compatible)?

1. Many rules satisfy a simple sufficient and necessary monotonicity condition.
2. All implementable rules are payoff-equivalent to randomization among ternary rules.

Answer

Short Version, Diagram

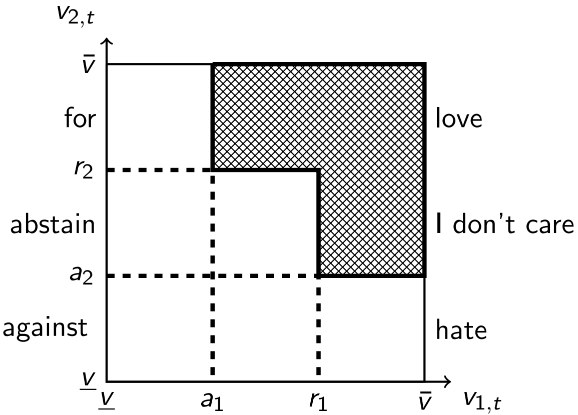
- Which allocation rules are implementable (incentive compatible)?



Answer

Short Version, Diagram, Other Labels

- Which allocation rules are implementable (incentive compatible)?



Agent

- Agent $i \in \{1, 2\}$ observes $v_{i,t} \in [\underline{v}, \bar{v}]$ in period t.

1. v_t are independent over time.
2. $v_{i,t}$ are possibly correlated between agents in each period.

- Agent i gets outside option $v_i^\star$ in period $T + 1$.

Principal

- Principal designs the allocation $q\left(v^t\right) \in \{0, 1\}$, stop or continue, given

1. the history of reports $v^{t-1} = v_1, v_2, ..., v_{t-1}$, and
2. the current report v_t.

Implementability

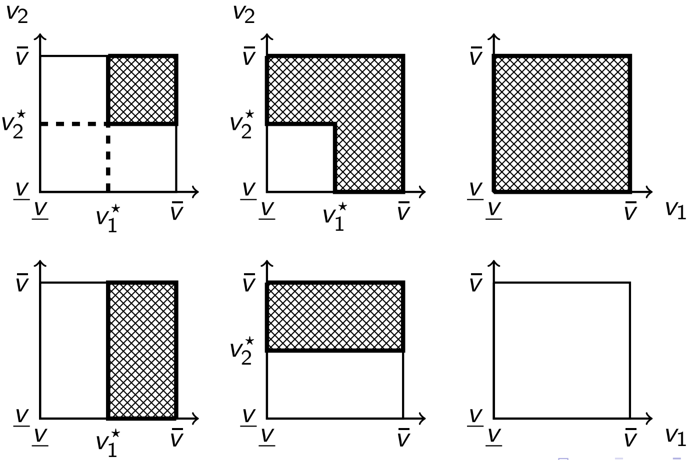
- Within-period ex-post incentive compatibility (wp-EPIC) is used.
- In every period, every agent prefers to report truthfully given everyone else's value.
- wp-EPIC is

1. robust to private communication,
2. robust to within period correlation,
3. robust to beliefs of the agents, and
4. tractable.

Static Implementability

Binary Mechanisms

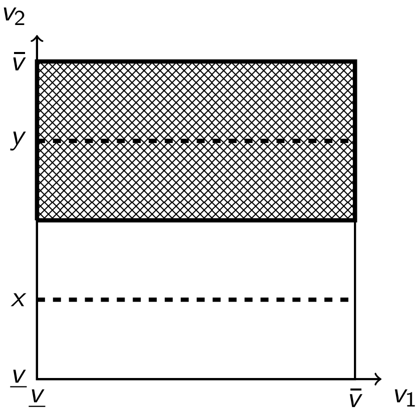
- There are six binary mechanisms: unanimity, reverse unanimity, 2 dictatorships, 2 constant mechanisms.



Static Implementability

Continuation Value, Constant

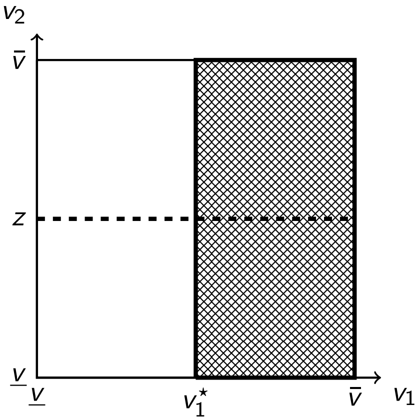
- Constant decisions are always incentive compatible.



Static Implementability

Continuation Value, Threshold

- A decision with threshold not equal to $v_1^\star$ is not incentive compatible for agent 1.



Static Implementability

Characterization

Lemma

If $T = 1$, a mechanism is incentive compatible iff it is binary.

Dynamic Mechanisms

- After each history v^{t-1}, there is one stage mechanism.
- The stage mechanism specifies, for each v_t,

1. whether $q = 0$ or $q = 1$ in this period, and
2. if $q = 0$, a continuation value that summarizes the sequence of stage mechanisms in the periods $t + 1, t + 2, \ldots T$.

Deterministic Mechanisms

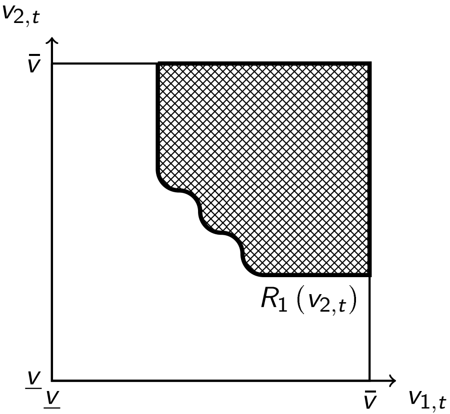
- Only "quasi-deterministic" mechanisms are considered, in which

1. every stage mechanism is deterministic, and
2. between-period randomization is allowed.

Dynamic Implementability

Monotonicity

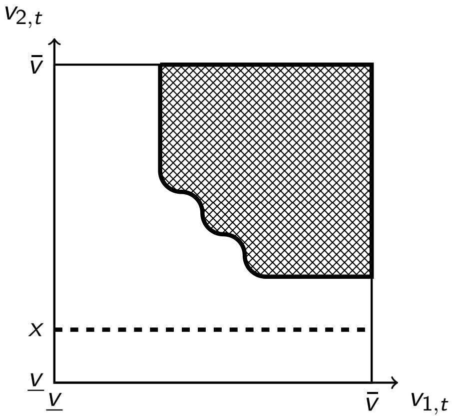
- Monotonicity implies that there is a threshold value above which $q = 1$ and below which $q = 0$.



Dynamic Implementability

Continuation Value, Constant

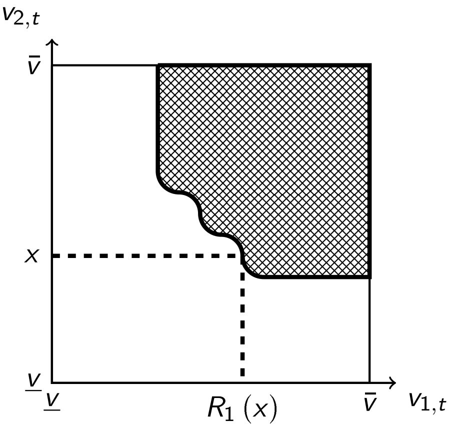
- Continuation value must be constant along $v_{2,t} = x$.



Dynamic Implementability

Continuation Value, Threshold 1

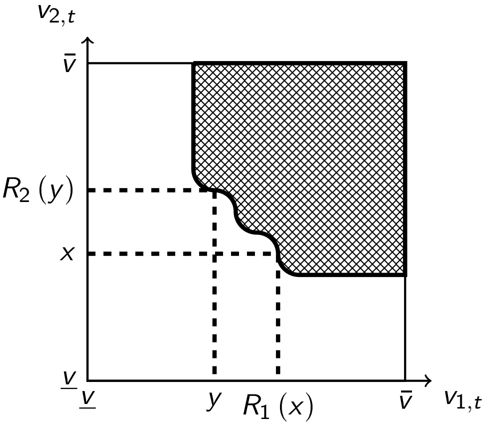
- Continuation value must be constant along $v_{2,t} = x$ and equal to the threshold value $R_1(x)$.



Dynamic Implementability

Continuation Value, Threshold 2

- Continuation value for the other player must be constant along $v_{1,t} = y$ and equal to the threshold value $R_2(y)$.



Dynamic Implementability

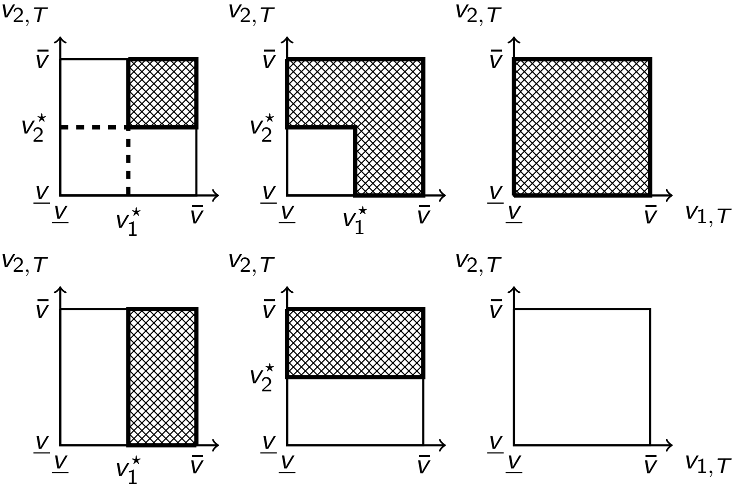
Lemma

A mechanism is incentive compatible iff all its stage mechanisms are

1. *monotonic in each $v_{i,t}$, and*
2. *continuation value for agent i is independent of $v_{i,t}$ and equal to the threshold value when it exists.*

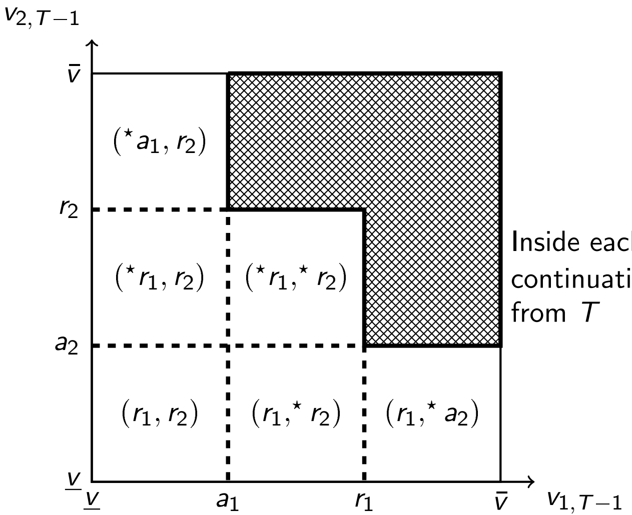
An Example

Last Period



An Example

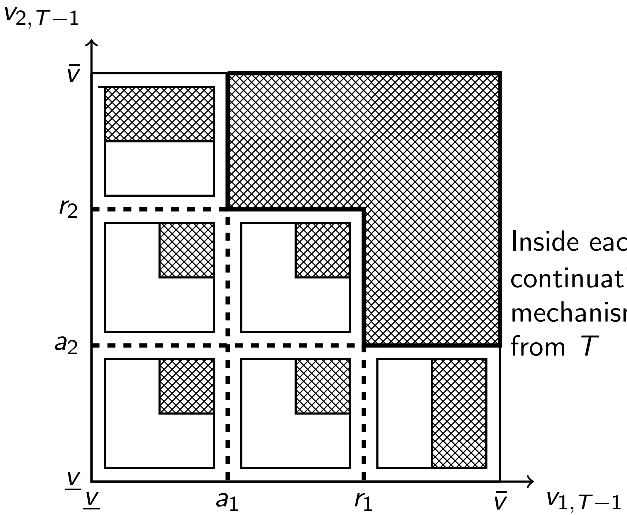
Second-to-Last Period, Continuation Value



Inside each region: continuation value from T

An Example

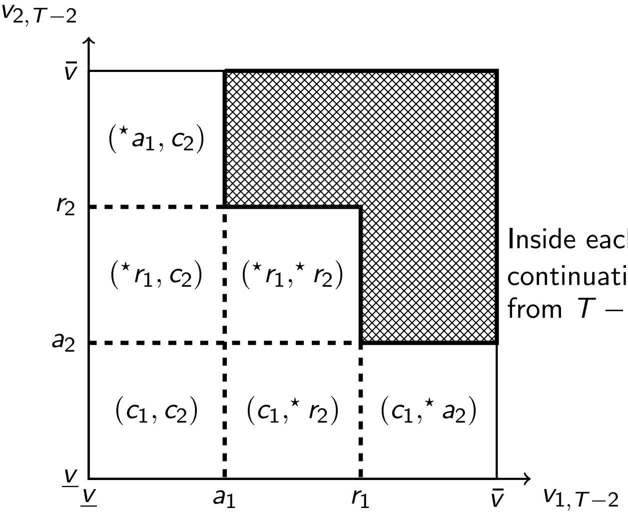
Second-to-Last Period



Inside each region: continuation mechanism from T

An Example

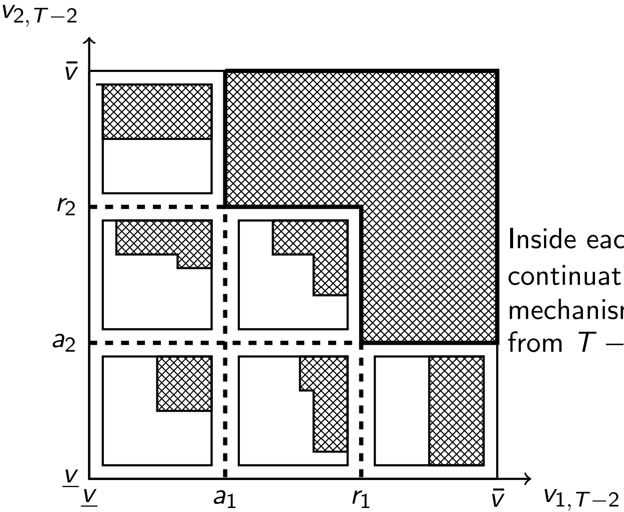
Third-to-Last Period, Continuation Value



Inside each region: continuation value from $T-1$

An Example

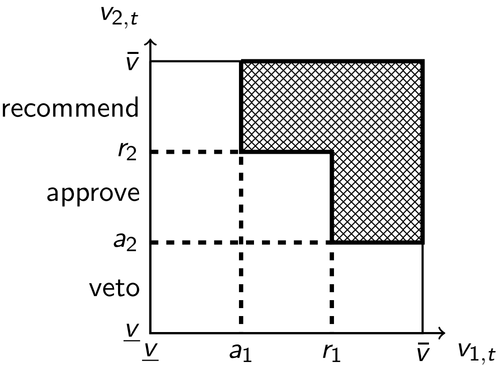
Third-to-Last Period



Inside each region: continuation mechanism from $T-1$

Ternary Mechanisms

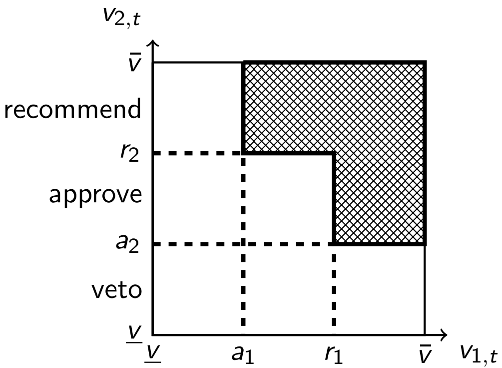
- The domain of each agent has three regions: veto, approve, recommend.
- $q = 1$ when no agent vetoes and at least one agent recommends.



Ternary Mechanisms

Relation to Binary Mechanisms

- All binary mechanisms are ternary with $a_i = r_i$ or $a_i = \underline{v}$ or $r_i = \bar{v}$.

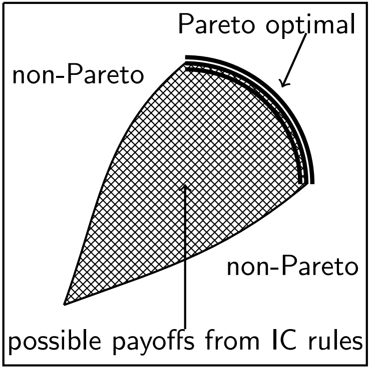


Pareto Optimality

- Pareto dominance is in terms of ex-ante expected continuation value at the beginning of a period.
- Pareto optimal means optimality among incentive compatible mechanisms.

Set of Incentive Compatible Continuation Values

- The set of continuation values that correspond to some incentive compatible mechanism is convex due to between-period randomization.



Pareto Optimal Mechanisms

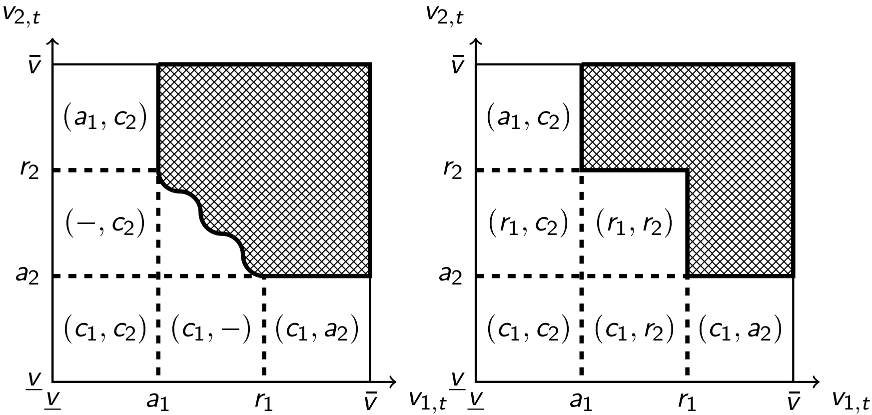
Lemma

Mechanisms on the Pareto boundary are payoff-equivalent to randomizations among ternary mechanisms.

Pareto Optimal Mechanisms

Proof

- The ternary mechanism on the right is better for both agents.



Non-Pareto Optimal Mechanisms

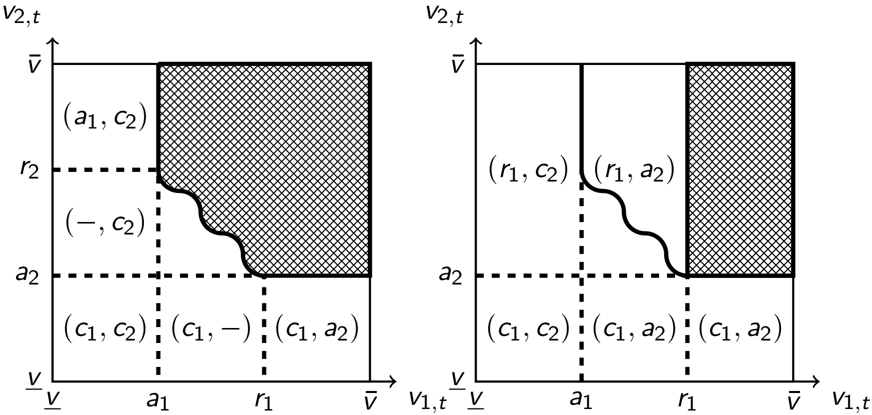
Lemma

Mechanisms on the non-Pareto optimal boundary are payoff-equivalent to randomizations among ternary mechanisms.

Non-Pareto Optimal Mechanisms

Proof

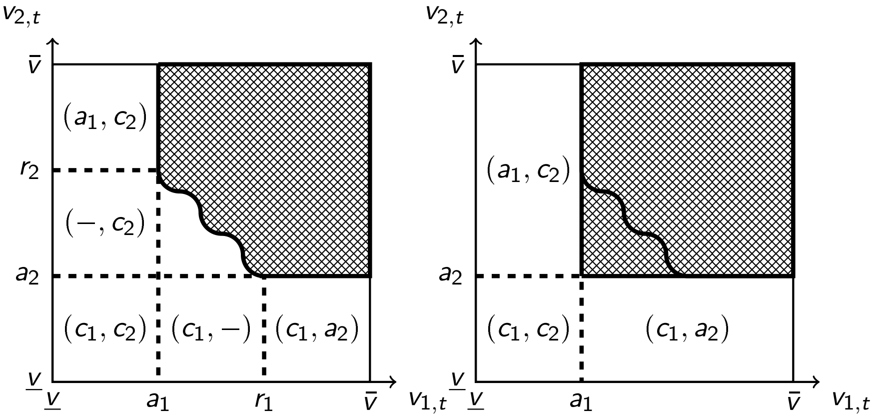
- The ternary mechanism on the right is better for agent 1 and worse for agent 2.



Non-Pareto Optimal Mechanisms

Proof

- The ternary mechanism on the right is worse for both agents.



Main Result

Theorem

Every incentive compatible mechanism is payoff-equivalent to a mechanism that is ternary in every stage.

Main Result

Implications

- Principal can restrict attention to only using ternary mechanisms.
- Optimal mechanism given any welfare function is ternary in every stage.

Related Literature

Dynamic Mechanism Design without Transfers

- Goods can be allocated in multiple periods in Guo and Horner (2015), Lipnowski and Ramos (2016).
- There is a single agent in Guo and Horner (2015), Kovac, Krahmer and Tatur (2013).
- There are multiple agents, but the good is allocated to one agent in Johnson (2014).

Related Literature

Committee Search

- Decision rules are restricted to,

1. unanimity rule in Moldovanu and Shi (2013), and
2. majority rule in Compt and Jehiel (2010) and Albrecht, Anderson and Vroman (2010).

Extension

Multiple Agents

- Mechanisms on the Pareto optimal boundary are still ternary.
- It is not clear whether mechanisms on the non-Pareto optimal boundary can be constructed from ternary mechanisms.

Extension

Random Mechanisms

- Any random mechanism can be decomposed into deterministic ones.
- It is not clear whether any random incentive compatible mechanism can be decomposed into deterministic incentive compatible ones.