Design of Search by Committees

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Story 1
Couple’s House

- A married couple looks for houses until they decide on purchasing one.
- The Canadian housing market is competitive and a house is gone before a new one becomes available.
Story 1

Couple’s House, Example: Sum Rule

- A married couple looks for houses until they decide on purchasing one.
- The Canadian housing market is competitive and a house is gone before a new one becomes available.

![Diagram](image)

- buy if $v_1 + v_2 > 14$

wife’s rating

husband’s rating

$v_1$

$v_2$
Story 1
Couple’s House, Example: Weighted Rules

- A married couple looks for houses until they decide on purchasing one.
- The Canadian housing market is competitive and a house is gone before a new one becomes available.

\[ v_1 + 2v_2 > 18 \]

wife important

\[ 2v_1 + v_2 > 10 \]

husband important
Story 1

Couple’s House, Example: Dictator Rules

- A married couple looks for houses until they decide on purchasing one.
- The Canadian housing market is competitive and a house is gone before a new one becomes available.

![Graph showing the division of preferences between wife and husband in a dictator scenario.](image)
Story 1
Couple’s House, Example: Ternary Rule

- A married couple looks for houses until they decide on purchasing one.
- The Canadian housing market is competitive and a house is gone before a new one becomes available.
Story 2
Committee Search, Example: Unanimity Rule

- A hiring committee receives job applications and conducts interviews until a position is filled.
- A decision is made right after each interview and is irreversible in small schools.
Story 3
Co-authors’ Research, Example: Reverse Unanimity Rule

- Co-authors gain access to new data sets periodically.
- The authors have differing opinions on whether a data set can lead to interesting results.
Committee search problems that have

1. sequential decision,
2. irreversible decision,
3. private value,
4. public allocation, and
5. no transfers.
Question

- Agents face stopping problem.
- Principal implements stopping rules.
- Which decision rules are implementable (incentive compatible)?
Answer

Short Version

- Which allocation rules are implementable (incentive compatible)?

1. Many rules satisfy a simple sufficient and necessary monotonicity condition.
2. All implementable rules are payoff-equivalent to randomization among ternary rules.
Which allocation rules are implementable (incentive compatible)?
Which allocation rules are implementable (incentive compatible)?
Agent

- Agent \( i \in \{1, 2\} \) observes \( v_{i,t} \in [v, \bar{v}] \) in period \( t \).

1. \( v_t \) are independent over time.
2. \( v_{i,t} \) are possibly correlated between agents in each period.

- Agent \( i \) gets outside option \( v_i^* \) in period \( T + 1 \).
Principal

- Principal designs the allocation $q(\nu^t) \in \{0, 1\}$, stop or continue, given

  1. the history of reports $\nu^{t-1} = \nu_1, \nu_2, \ldots, \nu_{t-1}$, and

  2. the current report $\nu_t$. 

Implementability

• Within-period ex-post incentive compatibility (wp-EPIC) is used.

• In every period, every agent prefers to report truthfully given everyone else’s value.

• wp-EPIC is

1. robust to private communication,
2. robust to within period correlation,
3. robust to beliefs of the agents, and
4. tractable.
Static Implementability

Binary Mechanisms

- There are six binary mechanisms: unanimity, reverse unanimity, 2 dictatorships, 2 constant mechanisms.
Constant decisions are always incentive compatible.
Static Implementability
Continuation Value, Threshold

- A decision with threshold not equal to $v_1^*$ is not incentive compatible for agent 1.
Static Implementability

Characterization

Lemma

If $T = 1$, a mechanism is incentive compatible iff it is binary.
Dynamic Mechanisms

- After each history \( v^{t-1} \), there is one stage mechanism.
- The stage mechanism specifies, for each \( v_t \),
  1. whether \( q = 0 \) or \( q = 1 \) in this period, and
  2. if \( q = 0 \), a continuation value that summarizes the sequence of stage mechanisms in the periods \( t + 1, t + 2, \ldots T \).
Deterministic Mechanisms

- Only “quasi-deterministic” mechanisms are considered, in which
  
  1. every stage mechanism is deterministic, and
  2. between-period randomization is allowed.
Dynamic Implementability

Monotonicity

- Monotonicity implies that there is a threshold value above which $q = 1$ and below which $q = 0$. 

![Diagram of monotonicity](image.png)
Dynamic Implementability
Continuation Value, Constant

- Continuation value must be constant along $v_{2,t} = \chi$. 

![Diagram](image-url)
Continuation value must be constant along $v_{2,t} = x$ and equal to the threshold value $R_1(x)$. 

\[ v_{2,t} \]

\[ \bar{v} \]

\[ x \]

\[ \bar{v} \]

\[ v \]

\[ v_{1,t} \]

\[ R_1(x) \]
Continuation value for the other player must be constant along $v_{1,t} = y$ and equal to the threshold value $R_2(y)$.
Dynamic Implementability

Lemma

A mechanism is incentive compatible iff all its stage mechanisms are

1. monotonic in each $v_{i,t}$, and

2. continuation value for agent $i$ is independent of $v_{i,t}$ and equal to the threshold value when it exists.
An Example

Last Period

\[ v_2, T \]

\[ v \]

\[ \bar{v} \]

\[ v_2^* \]

\[ v_1^* \]

\[ v_1, T \]
An Example
Seconds-to-Last Period, Continuation Value

Inside each region: continuation value from $T$
An Example

Second-to-Last Period

Inside each region: continuation mechanism from $T$
An Example

Third-to-Last Period, Continuation Value

Inside each region: continuation value from $T - 1$
An Example
Third-to-Last Period

Inside each region: continuation mechanism from $T - 1$
Ternary Mechanisms

- The domain of each agent has three regions: veto, approve, recommend.
- \( q = 1 \) when no agent vetoes and at least one agent recommends.
Ternary Mechanisms
Relation to Binary Mechanisms

- All binary mechanisms are ternary with $a_i = r_i$ or $a_i = v$ or $r_i = \bar{v}$.
Pareto Optimality

- Pareto dominance is in terms of ex-ante expected continuation value at the beginning of a period.
- Pareto optimal means optimality among incentive compatible mechanisms.
Set of Incentive Compatible Continuation Values

- The set of continuation values that correspond to some incentive compatible mechanism is convex due to between-period randomization.
Pareto Optimal Mechanisms

Lemma

Mechanisms on the Pareto boundary are payoff-equivalent to randomizations among ternary mechanisms.
Pareto Optimal Mechanisms

Proof

- The ternary mechanism on the right is better for both agents.
Non-Pareto Optimal Mechanisms

Lemma

Mechanisms on the non-Pareto optimal boundary are payoff-equivalent to randomizations among ternary mechanisms.
Non-Pareto Optimal Mechanisms

Proof

- The ternary mechanism on the right is better for agent 1 and worse for agent 2.
Non-Pareto Optimal Mechanisms

Proof

- The ternary mechanism on the right is worse for both agents.
Main Result

Theorem

*Every incentive compatible mechanism is payoff-equivalent to a mechanism that is ternary in every stage.*
Main Result

Implications

- Principal can restrict attention to only using ternary mechanisms.
- Optimal mechanism given any welfare function is ternary in every stage.
Related Literature
Dynamic Mechanism Design without Transfers

- Goods can be allocated in multiple periods in Guo and Horner (2015), Lipnowski and Ramos (2016).
- There is a single agent in Guo and Horner (2015), Kovac, Krahmer and Tatur (2013).
- There are multiple agents, but the good is allocated to one agent in Johnson (2014).
Related Literature

Committee Search

- Decision rules are restricted to,

1. unanimity rule in Moldovanu and Shi (2013), and
2. majority rule in Compt and Jehiel (2010) and Albrecht, Anderson and Vroman (2010).
Mechanisms on the Pareto optimal boundary are still ternary.

It is not clear whether mechanisms on the non-Pareto optimal boundary can be constructed from ternary mechanisms.
Any random mechanism can be decomposed into deterministic ones.

It is not clear whether any random incentive compatible mechanism can be decomposed into deterministic incentive compatible ones.