Ice Melting Rate Experiment

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Introduction

The purpose of this experiment is to **determine which factors have the greatest effect on the melting time of an ice cube**. Ice melts at different rates depending on environmental conditions, and understanding these influences can provide insights into heat transfer, thermodynamics, and practical considerations such as food handling or cooling processes.

This study focuses on **three key factors** believed to influence melting rate:

- **surface material**: different surfaces conduct heat at different rates
- **ambient temperature**: the surrounding environment affects heat transfer to the cube
- **salt presence**: salt lowers the freezing point of water and may speed melting

The response variable is **total time (minutes)** for a standard ice cube to melt completely.

Materials and Equipments

- Ice cubes from same tray (1 in × 1 in)
- Metal baking sheet (4 in × 4 in)
- Plastic cutting board (4 in × 4 in)
- Standard table surface (room temperature environment)
- Refrigerator
- Timer
- Salt (0.5 g per salted treatment)
- Optical thermometer
- Paper towels (surface drying)

Equipment Factors and Levels

Table 1: Equipment Factors and Levels

Factor	Low Level (-)	High Level (+)	Description
Surface Material (A)	Plastic	Metal	Different thermal conductivities
Temperature (B)	Fridge (4°C)	Room (22°C)	Surrounding ambient temperature
Salt Presence (C)	No salt	0.5 g salt added	Alters freezing/melting behavior

Experimental Procedure

For each trial:
1. Pre-condition the assigned surface material in its proper environment (15 minutes).

2. Place the surface flat on the testing location.
3. Place one ice cube onto the surface and start the timer immediately.
4. Observe the cube until no solid ice remains.
5. Record the melting time to the nearest minutes.
6. Wipe and fully dry all surfaces between trials to maintain consistent starting conditions.

Ambient temperature will be confirmed using an optical thermometer.

Experimental Design

A 2^3 full factorial design will be used, allowing simultaneous investigation of all main effects and interactions among the three factors

- Number of factors: 3
- Levels per factor: 2
- Total treatment combinations: $2^3 = 8$
- Replicates: 3 per combination
- Total trial: 24
- Run order will be randomized within each replication to reduce bias.

Table 2: Model Matrix

Run	A	B	C	Surface	Temperature	Salt	y1	y2	y3	ybar	s^2	$\ln(s^2)$
1	-	-	-	Plastic	Fridge (4°C)	No salt	305	312	308	308.33333	12.333333	2.5123056
2	+	-	-	Metal	Fridge (4°C)	No salt	302	300	297	299.66667	6.333333	1.8458267
3	-	+	-	Plastic	Room (22°C)	No salt	78	79	75	77.33333	4.333333	1.4663371
4	+	+	-	Metal	Room (22°C)	No salt	60	62	63	61.66667	2.333333	0.8472979
5	-	-	+	Plastic	Fridge (4°C)	0.5 g salt	316	329	321	322.00000	43.000000	3.7612001
6	+	-	+	Metal	Fridge (4°C)	0.5 g salt	311	309	315	311.66667	9.333333	2.2335922
7	-	+	+	Plastic	Room (22°C)	0.5 g salt	80	83	81	81.33333	2.333333	0.8472979
8	+	+	+	Metal	Room (22°C)	0.5 g salt	66	63	68	65.66667	6.333333	1.8458267

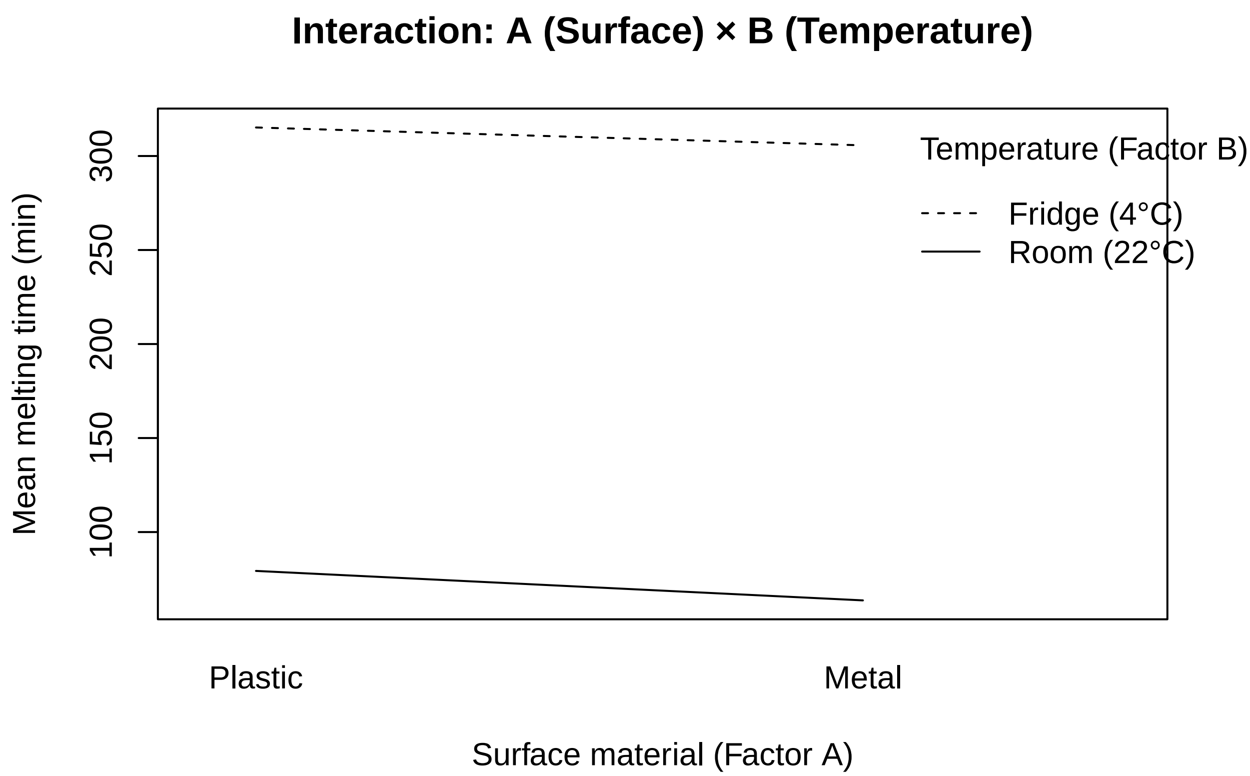
Statistical Analysis Plan

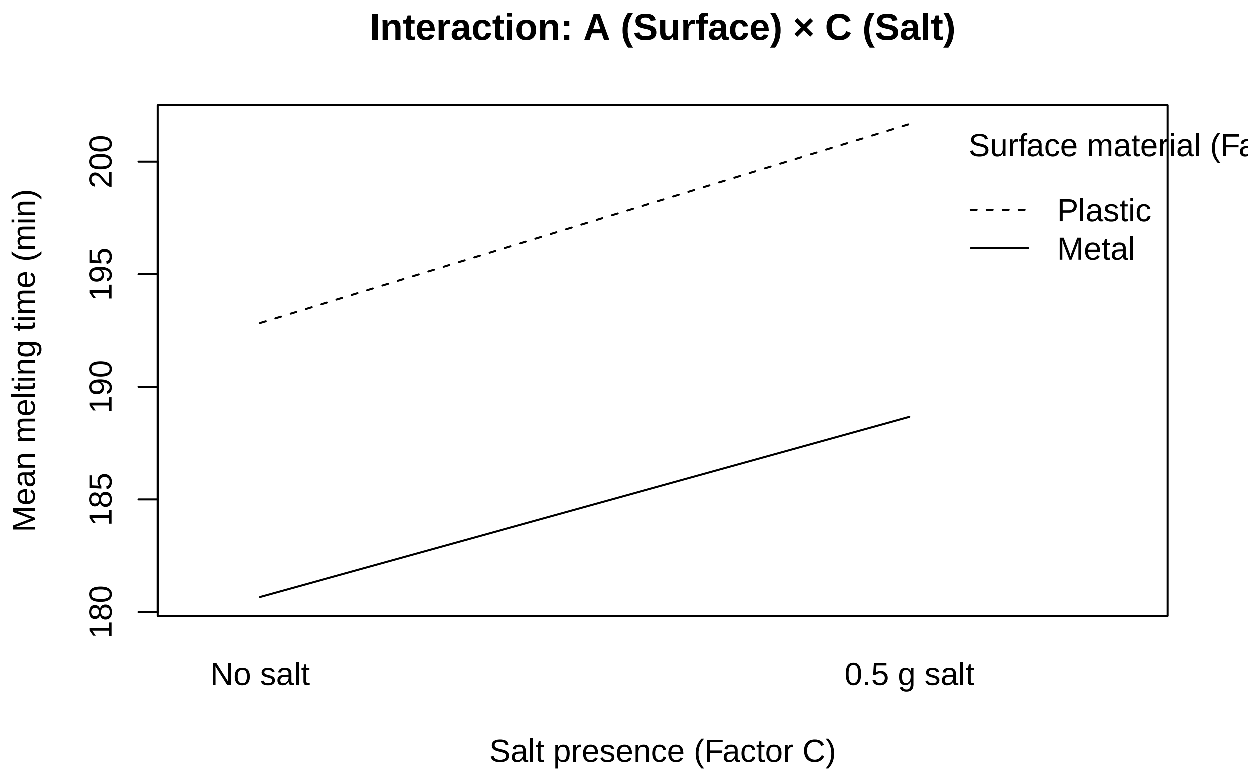
1. Interaction Effect Plots First, I will visualize how pairs of factors jointly affect mean melting time using interaction plots:

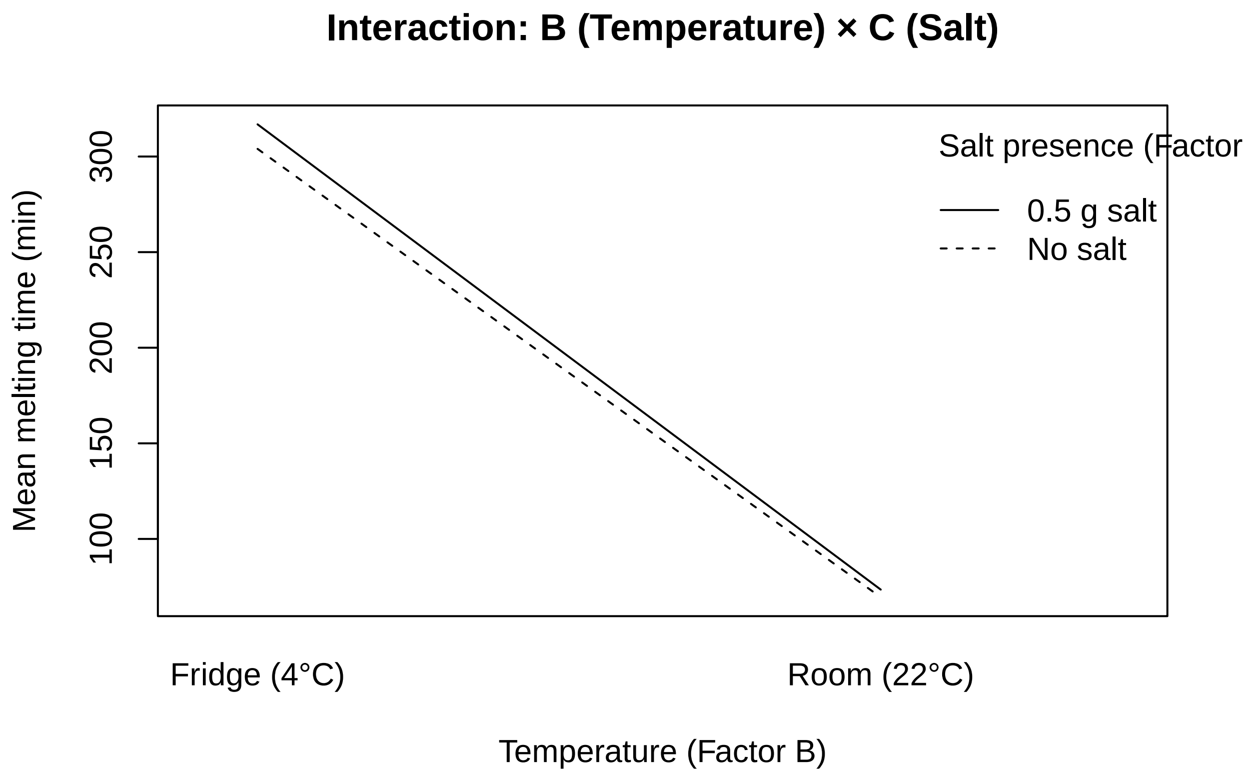
- Compute the mean melting time for each treatment combination.

- For each pair of factors (A–B, A–C, B–C), create interaction plots where:

- The x-axis is the level of one factor $(-, +)$.
- Separate lines show the levels of the second factor $(-, +)$.




Interaction: A (Surface) × C (Salt)
Surface material (Fa
Plastic
Metal
Mean melting time (min)
180
185
190
195
200
No salt
0.5 g salt
Salt presence (Factor C)



Key Observations: The interaction plots show that **temperature (factor B) has by far the largest impact on melting time**. Across all runs, the average melting time at fridge temperature (B = −) is about 310 minutes, while at room temperature (B = +) it drops to about 72 minutes, a difference of roughly 239 minutes.

Surface material (A) has a smaller but consistent effect: metal (+) yields shorter melt times than plastic (−) by about 13 minutes on average. **Salt (C) produces only a modest effect**, with salted runs being slightly slower (about 8 minutes longer on average), which is opposite to the usual intuition but consistent across all combinations. The interaction lines are nearly parallel, indicating weak interactions among A, B, and C; the overall pattern is dominated by the main effect of temperature.

2. Half-Normal Plot of Effects Second, I will construct a half-normal plot of the factorial effects to visually identify large effects:

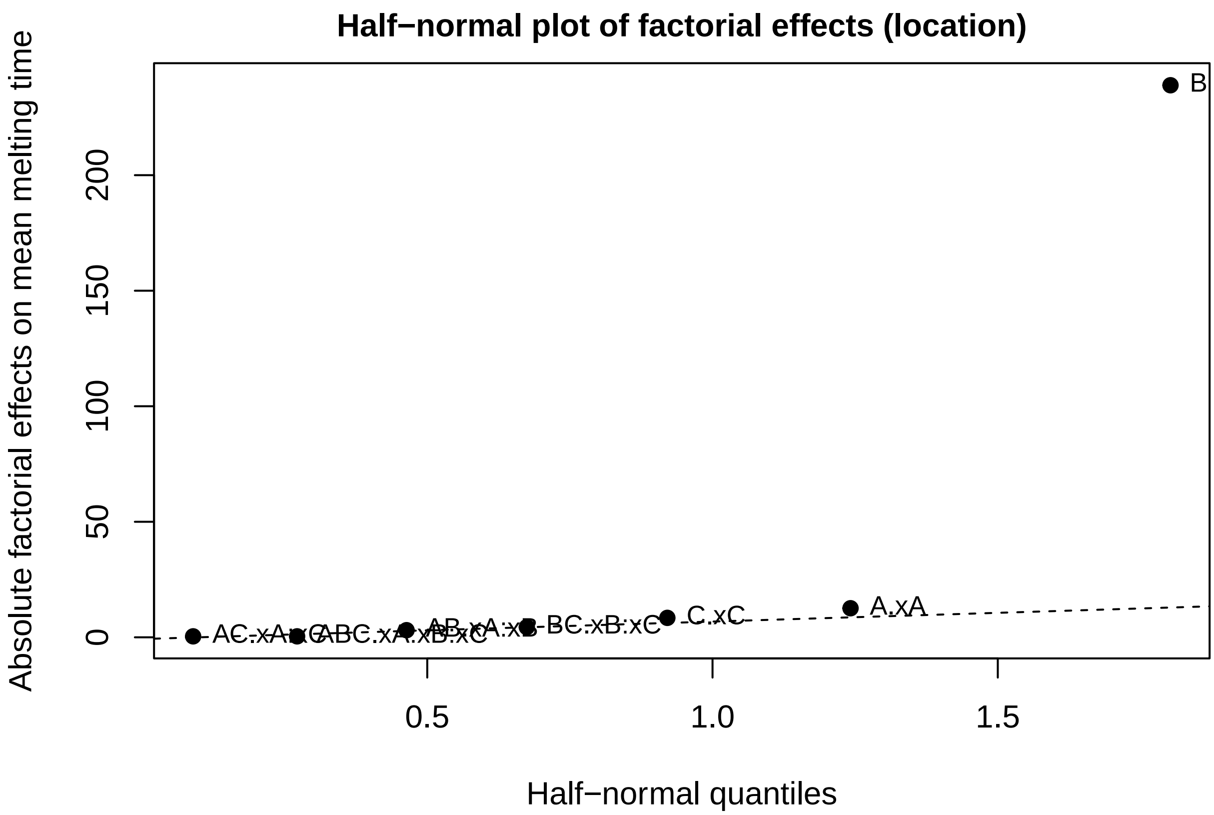
- Code factors A, B, C at levels −1 and +1.
- Fit the full 2^3 model and compute factorial effects for A, B, C, AB, AC, BC, and ABC.
- Take absolute values of these effects and sort them from smallest to largest.
- Plot the sorted absolute effects against theoretical half-normal quantiles.

Table 3: Regression coefficients from the coded 2^3 model for mean melting time.

	Term	Coefficient	Explanation
(Intercept)	(Intercept)	190.958	Overall mean under coded design (ˆ)
xA	xA	-6.292	Main effect contrast for surface (A)
xB	xB	-119.458	Main effect contrast for temperature (B)
xC	xC	4.208	Main effect contrast for salt (C)
xA:xB	xA:xB	-1.542	AB interaction contrast
xA:xC	xA:xC	-0.208	AC interaction contrast
xB:xC	xB:xC	-2.208	BC interaction contrast
xA:xB:xC	xA:xB:xC	0.208	ABC three-way interaction contrast

Table 4: Factorial effects (2 × regression coefficients) for the 2^3 design.

Effect	Effect.value	Explanation
A.xA	-12.583	Estimated main effect of surface (A)
B.xB	-238.917	Estimated main effect of temperature (B)
C.xC	8.417	Estimated main effect of salt (C)
AB.xA:xB	-3.083	Estimated AB interaction effect
AC.xA:xC	-0.417	Estimated AC interaction effect
BC.xB:xC	-4.417	Estimated BC interaction effect
ABC.xA:xB:xC	0.417	Estimated ABC three-way interaction effect



Key Observations: In this plot, **the absolute effect for B (temperature) stands out clearly away from the approximate straight line** formed by the smaller effects. **All other effects (A, C, and the interactions) lie close to that line** and are much smaller in magnitude. This visual evidence indicates that B is the only clearly dominant effect on melting time, while A, C, and all interactions are comparatively minor.

3. Lenth's Method (Formal Screening of Effects) Third, I will use Lenth's method as a formal, effect-based significance test:

- Use the absolute effect estimates from the full factorial model.
- Compute the pseudo standard error (PSE) from the median of the smaller effects (excluding unusually large ones).
- For each effect, compute a t-like statistic: effect / PSE.
- Compare |effect| or |t-like| values to Lenth's critical thresholds (e.g., using the Individual Error Rate).
- Classify effects as significant or not, based on these thresholds.

Table 5: Lenth's method for mean melting time: effect estimates, |t_PSE|, and significance using IER = 0.05 (critical value 2.166).

	Effect	Estimate	AbsEffect	X.t_PSE.	Significant..IER.0.05.
B.xB	B.xB	-238.917	238.917	42.474	Yes
A.xA	A.xA	-12.583	12.583	2.237	Yes
C.xC	C.xC	8.417	8.417	1.496	No
BC.xB:xC	BC.xB:xC	-4.417	4.417	0.785	No
AB.xA:xB	AB.xA:xB	-3.083	3.083	0.548	No
AC.xA:xC	AC.xA:xC	-0.417	0.417	0.074	No
ABC.xA:xB:xC	ABC.xA:xB:xC	0.417	0.417	0.074	No

For this 2^3 design we have $I = 7$ effects. Using the IER critical value $c = 2.166$, an effect is significant if $|t_{\mathrm{PSE}}| > 2.166$.

Key Observations: Under Lenth's screening framework, **B is unambiguously significant**, while the other effects are small and would not be classified as important at usual significance levels; **A is at borderline, barely above the threshold. This formal result agrees with the half-normal plot: temperature is the only clearly active factor, while the remaining effects, including A, are borderline or negligible in practical terms.**

Conclusion

This experiment investigated how **surface material (A), ambient temperature (B), and salt presence (C)** affect the melting time of an ice cube using a full 2^3 factorial design with three replicates per condition. Across all analyses, the results were consistent:

- **Temperature (factor B) is overwhelmingly the dominant factor** influencing melting time.

- Surface material (factor A) shows a small but statistically detectable effect.
- Salt presence (C) and all interaction effects (AB, AC, BC, ABC) are small and not statistically significant.

Future work

Several extensions could deepen the understanding of ice-melting behavior:

- Refine the effect of surface material (A): Since A was statistically detectable but small, testing a wider variety of surfaces with different thermal conductivities (e.g., glass, wood, aluminum, steel) could reveal whether the small effect persists or becomes more meaningful with more diverse materials.
- Model melting time as a function of temperature: adding more levels (e.g., 0°C, 10°C, 30°C) would allow fitting curvature and developing a predictive model rather than only a high/low contrast.
- Investigate nonlinear or threshold effects of salt: The binary 0 g vs. 0.5 g salt comparison may be too coarse. A full range of concentrations might show a clearer trend.
- Study environmental variables like airflow or humidity: These can alter convection and evaporation, potentially interacting with temperature or surface properties.

Appendix: Key R Code Summary

```
planning <- data.frame(
  Run = 1:8,
  A   = c("-", "+", "-", "+", "-", "+", "-", "+"),
  B   = c("-", "-", "+", "+", "-", "-", "+", "+"),
  C   = c("-", "-", "-", "-", "+", "+", "+", "+")
)
planning$Surface     <- ifelse(planning$A == "-", "Plastic", "Metal")
planning$Temperature <- ifelse(planning$B == "-", "Fridge (4°C)", "Room (22°C)")
planning$Salt        <- ifelse(planning$C == "-", "No salt", "0.5 g salt")
```

Experimental Design Matrix

```
real_data <- planning |>
  mutate(
    y1 = c(...),
    y2 = c(...),
    y3 = c(...)
  ) |>
  rowwise() |>
  mutate(
    ybar  = mean(c(y1, y2, y3)),
    s2    = var(c(y1, y2, y3)),
    ln_s2 = log(s2)
  ) |>
  ungroup()
```

Data Entry and Summary Statistics

```
interaction.plot(
  x.factor     = plot_data$A,
  trace.factor = plot_data$B,
  response     = plot_data$ybar
)
```

Interaction plot

```
coded_data <- real_data |>
  mutate(
    xA = ifelse(A == "-", -1, 1),
    xB = ifelse(B == "-", -1, 1),
```

```
    xC = ifelse(C == "-", -1, 1),
    xAB   = xA * xB,
    xAC   = xA * xC,
    xBC   = xB * xC,
    xABC = xA * xB * xC
  )
```

Factor Coding for Model Fitting

```
fit_mean <- lm(ybar ~ xA * xB * xC, data = coded_data)
coefs <- coef(fit_mean)
effects_y <- c(
  A   = 2 * coefs["xA"],
  B   = 2 * coefs["xB"],
  C   = 2 * coefs["xC"],
  AB  = 2 * coefs["xA:xB"],
  AC  = 2 * coefs["xA:xC"],
  BC  = 2 * coefs["xB:xC"],
  ABC = 2 * coefs["xA:xB:xC"]
)
```

Full 2^3 Regression Model and Effects

```
effects_abs <- sort(abs(effects_y))
half_q <- qnorm(0.5 + 0.5 * (i - 0.5) / I)

plot(half_q, effects_abs)
abline(fit_ref, lty = 2)
text(half_q, effects_abs, labels = names(effects_abs))
```

Half-Normal Plot of Effects

```
lenth_table <- function(effects, ier_crit = 2.166) { ... }
lenth_mean <- lenth_table(effects_y)
```

Lenth's Method for Effect Screening